This exam is to be done WITHOUT a calculator. Be sure to clearly and neatly show your work where appropriate. Provide enough detail to support your answer. Clearly indicate your answer by circling it.

1. Perform the operations indicated below. Use order of operations rules and reduce any rational numbers to lowest terms.

\[
\begin{align*}
(a) \quad \frac{3}{8} \div \frac{15}{14} &= \frac{7}{20} \\
(b) \quad \frac{5}{12} + \frac{9}{10} &= \frac{29}{60} \\
(c) \quad \frac{63}{50} \cdot \frac{33}{49} &= \frac{297}{175} \\
(d) \quad -3 + (+14) &= 11 \\
(e) \quad (-2)(-9)(-1) &= -18 \\
(f) \quad 3 + 4 \cdot 2 &= 11 \\
(g) \quad (3 + 4) \cdot 2 &= 14 \\
(h) \quad (2^2 - 1)^3 &= 27 \\
(i) \quad 100 \div 5 \cdot 2 &= 40 \\
(j) \quad (2^2 - 3^2)^2 - (2 + 3)^2 \\
\quad (4 - 9)^2 - (5)^2 \\
\quad (-5)^2 - 25 \\
\quad 25 - 25 &= 0 \\
(k) \quad \frac{3 + 4(-3)}{3 - 4(-3)} &= \frac{-3}{10} \\
(l) \quad \frac{5 - 3^3}{5 - 3^3} &= -\frac{4}{11}
\end{align*}
\]

2. Insert one of the symbols: "<" or ">") to make a true statement.

\[
\begin{align*}
(a) \quad -10 \quad \text{<} \quad -3 \\
(b) \quad 5 \quad \text{>} \quad -1000
\end{align*}
\]
3. Simplify each expression as much as possible. Use the distributive property where appropriate and combine like terms.

(a) \(3 + 5x + 8 + 3x\)
\[
= 3 + (-8) + 5x + (-3x)
\]
\[
= -5 + 2x
\]

(b) \(2(4 - 2x) + 5x\)
\[
= 8 - 4x + 5x
\]
\[
= 8 + x
\]

(c) \(6xy + 3x - y + 4(x - xy)\)
\[
= 6xy + 3x - y + 4x - 4xy
\]
\[
= 6xy + 3x - y + 4x + (-4xy)
\]
\[
= 6xy + (-4xy) + 3x + 4x - y
\]
\[
= 2xy + 7x - y
\]

(d) \(2(x^2 + 3x) - 3(x - x^2)\)
\[
= 2x^2 + 6x + (-3x) + (4x^2)
\]
\[
= 2x^2 + 3x^2 + 6x + (-3x)
\]
\[
= 5x^2 + 3x
\]

4. State the property of real numbers illustrated below

(a) \(a + b + c = c + a + b\)
COMMUTATIVE PROP. OF ADDITION

(b) \(3 \cdot (5x) = (3 \cdot 5)x\)
ASSOCIATIVE PROP. OF MULTIPLICATION

5. Two people are painting a large wall. The first person paints 3 square feet per minute while the second person paints 4 square feet per minute.

(a) Let \(x\) be the number of minutes they have been painting. Write an expression that represents the total number of square feet painted by both painters in \(x\) minutes.

\[3x\] IS NUMBER OF SQ. FT. PAINTED BY FIRST PAINTER IN \(x\) MINS.

\[4x\] IS NUMBER OF SQ. FT. PAINTED BY SECOND PAINTER IN \(x\) MINS.

**TOTAL IS** \(3x + 4x\) OR \(7x\)

(b) Find the number of square feet painted in one hour.

ONE HOUR IS 60 MINUTES.

SO THEY PAINT **720 SQ. FT.** IN 1 HOUR.

(c) How many minutes would it take them to paint a wall 7 feet tall by 10 feet wide?

7 FEET TALL BY 10 FEET WIDE IS AN AREA OF 70 SQ. FT.

For \(7x\) to be equal to 70, \(x\) would have to be 10.

So it would take them **10 MINUTES**
Work the problems below. Please provide sufficient details to support your answers.

1. Solve each of the equations below using the properties of equality and the properties of the real numbers. Write your solutions in a solution set.

   (a) \(-3x - 5 = 16\)
   \[\Rightarrow -3x = 21\]
   \[\Rightarrow x = -7\]
   \[\text{Solution: } x = -7\]

   (b) \(2(3x - 5) - x = -30\)
   \[\Rightarrow 6x - 10 - x = -30\]
   \[\Rightarrow 5x = -20\]
   \[\Rightarrow x = -4\]
   \[\text{Solution: } x = -4\]

   (c) \(5x - 2 = 2(x - 1) + 6\)
   \[\Rightarrow 5x - 2 = 2x - 2 + 6\]
   \[\Rightarrow 3x = 6\]
   \[\Rightarrow x = 2\]
   \[\text{Solution: } x = 2\]

   (d) \(\left(\frac{2}{3}x - \frac{2}{3}\right) \cdot 18\)
   \[\Rightarrow 4x - 12 = 15\]
   \[\Rightarrow 4x = 27\]
   \[\Rightarrow x = \frac{27}{4}\]
   \[\text{Solution: } x = \frac{27}{4}\]

2. Solve each of the inequalities below using the properties of inequality and the properties of the real numbers. Write your solution sets using set-builder notation and graph the solution set on a number line.

   (a) \(3x + 7 > 5\)
   \[\Rightarrow 3x > -2\]
   \[\Rightarrow x > -\frac{2}{3}\]
   \[\text{Solution: } x > -\frac{2}{3}\]

   (b) \(x - 3(x + 5) \leq 7\)
   \[\Rightarrow x - 3x - 15 \leq 7\]
   \[\Rightarrow -2x - 15 \leq 7\]
   \[\Rightarrow -2x \leq 22\]
   \[\Rightarrow x \geq -11\]
   \[\text{Solution: } x \geq -11\]

3. Given the formula \(z = y - 3xy\),

   (a) Find the value of \(z\) if \(x = -2\) and \(y = 5\).
   \[z = 5 - 3(-2)5 = 5 - (-30)\]
   \[= 5 + 30 = 35\]

   (b) Solve the formula for the variable \(x\).
   \[z - y = -3xy \Rightarrow x = \frac{z - y}{-3y}\]
   \[\text{or } x = \frac{y - z}{3y}\]
4. Sketch as accurately as possible the graphs of each linear equation in the coordinate system to the right. Label each line with the letter corresponding to its equation.

(a) \( x = 3 \)
(b) \( y = -1 \)
(c) \( x + 2y = 4 \)
(d) \( y = \frac{2}{3}x - 2 \)

5. What are the slope and \( y \)-intercept of the line with equation \( y = \frac{2}{3}x - 2 \)?

\[
\begin{align*}
\text{Slope} & = \frac{2}{3} \\
\text{Y-intercept} & = -2
\end{align*}
\]

6. What is the point-slope equation of the line whose slope is -2 and which goes through the point \((-3, 1)\)?

\[
\begin{align*}
y - 1 & = -2(x + 3) \\
y & = -2x - 7
\end{align*}
\]

7. What is the slope-intercept equation of the line which goes through the points \((2, 3)\) and \((1, 7)\)?

\[
\begin{align*}
m & = \frac{\Delta y}{\Delta x} = \frac{7 - 3}{1 - 2} = \frac{4}{-1} = -4 \\
y & = -4x + b \\
3 & = -4.2 + b \\
3 & = -8 + b \\
b & = 11
\end{align*}
\]

\[
y = -4x + 11
\]

8. What is the slope-intercept equation of the line which goes through the point \((0, -2)\) and is perpendicular to the line with equation \( y = \frac{2}{3}x - 8 \)?

\[
\begin{align*}
\text{Perp. to line w. slope } \frac{3}{5} \text{ so slope } m & = -\frac{5}{3} \\
\text{Goes through } (0, -2) \text{ so } b & = -2
\end{align*}
\]

\[
y = -\frac{5}{3}x - 2
\]
9. The population of the United States in the year 1960 was 179 million. In the year 2000 the population was 281 million. Let \( t \) be the number of years after 1960, let \( P \) be the population in millions. The pair \((t, P)\) gives the population \( P \) (in millions) at at time \( t \) years after 1960. Assume the population is linearly related to the year.

(a) Find the slope of the line containing the points \((0, 179)\) and \((40, 281)\) and interpret this number in the context of this problem (that is, what does this number tell you about how the population is changing with time?)

\[
\begin{align*}
\frac{281-179}{40-0} &= \frac{102}{40} \text{ million ppl} \\
&= \frac{2.55}{1} \text{ million ppl per year}
\end{align*}
\]

(b) By how much will the population increase in any 10 year time period?

\[
\Delta P = m \Delta t \\
= (2.55) \times 10 \text{ million ppl} \ 	ext{ or } \ 25.5 \text{ million ppl}
\]

10. Suppose a triangle has angles such that the largest angle is five times the smallest angle and the other angle is three times the smallest angle. Let \( x \) be the measure of the smallest angle. Set up and solve an equation to find the measures of all three angles.

\[
\begin{align*}
x &= \text{SMALLEST ANGLE} \\
5x &= \text{LARGEST ANGLE} \\
3x &= \text{MID-SIZED ANGLE}
\end{align*}
\]

\[
x + 5x + 3x = 180 \\
9x = 180 \Rightarrow x = 20
\]

The angles have measures \( 20^\circ, 60^\circ, 100^\circ \)
Work the problems below. Please provide sufficient details to support your answers.

1. Simplify (reduce) each rational expression.

   (a) \[
   \frac{x^2 + 6x + 9}{x^2 - 9} = \frac{(x+3)(x+3)}{x+3} = \frac{x+3}{x-3}
   \]

   (b) \[
   \frac{6x^2 + x - 12}{6x^2 + 9x} = \frac{(2x+3)(3x-4)}{3x(2x+3)} = \frac{3x-4}{3x}
   \]

   (c) \[
   \frac{x^2 - 3x}{2x - 6} = \frac{x(x-3)}{2(x-3)} = \frac{x}{2}
   \]

   (d) \[
   \frac{x^2 + wx + xt + wt}{xr - xs + rt - st} = \frac{x(x+w) + t(x+w)}{x(r-s) + t(r-s)} = \frac{(x+w)(x+t)}{(r-s)(x+t)}
   \]

2. Multiply or divide as indicated. Be sure to try to reduce your answers.

   (a) \[
   \frac{x^2 + 6x + 5}{x^2 - 25} \cdot \frac{x^2 - 5x}{x^2 - x - 2}
   \]

   (b) \[
   \frac{x + 1}{x - 1} \div \frac{2x^2 + x - 1}{x^2 - 2x + 1}
   \]

   \[
   \frac{x + 1}{x - 1} \cdot \frac{(x-1)(x+1)}{(2x-1)(x+1)} = \frac{x-1}{2x-1}
   \]
3. Add or subtract as indicated. Be sure to try to reduce your answers.

(a) \[ \frac{x^2 - 1}{2x + 1} + \frac{x^2 - x}{2x + 1} \]

\[ = \frac{2x^2 - x - 1}{2x + 1} = \frac{(2x+1)(x-1)}{2x+1} \]

\[ = x - 1 \]

(b) \[ \frac{x}{x^2 - 1} - \frac{1}{2x + 2} \]

\[ = \frac{2}{2} \cdot \frac{x}{(x+1)(x-1)} - \frac{1}{2} \cdot \frac{(x-1)}{(x+1)(x-1)} \]

\[ = \frac{2x - x + 1}{2(x+1)(x-1)} = \frac{(x+1)}{2(x+1)(x-1)} \]

\[ = \frac{1}{2(x-1)} \]

4. Simplify each complex fraction be sure your answer is completely reduced.

(a) \[ \frac{x}{x^2 - 1} \]

\[ \frac{1}{x + 1} \]

\[ = \frac{\sqrt{x}}{(x+1)(x-1)} \cdot \frac{(x+1)}{(x+1)} \]

\[ = \frac{x}{x - 1} \]

(b) \[ \frac{\left( \frac{1}{x} + \frac{1}{y} \right) x^2 y^2}{\left( \frac{1}{x^2} - \frac{1}{y^2} \right) x^2 y^2} \]

\[ = \frac{xy^2 + x^2 y}{y^2 - x^2} \]

\[ = \frac{xy (y+x)}{(y-x)(y+x)} \]

\[ = \frac{xy}{y-x} \]
5. Solve each equation below.

(a) $5x(x - 2)(3x + 5) = 0$

$$\Rightarrow x = 0 \text{ or } x - 2 = 0 \text{ or } 3x + 5 = 0$$

$$\Rightarrow x = 0 \text{ or } x = 2 \text{ or } x = -\frac{5}{3}$$

$$S = \{0, 2, -\frac{5}{3}\}$$

(b) $2x^2 + 3x - 2 = 0$

$$\Rightarrow (2x + 1)(x + 2) = 0$$

$$\Rightarrow 2x + 1 = 0 \text{ or } x + 2 = 0$$

$$\Rightarrow x = -\frac{1}{2} \text{ or } x = -2$$

$$S = \{-\frac{1}{2}, -2\}$$

(c) $\frac{1}{z - 2} = \frac{z}{3}$

$$\Rightarrow z^2 - 2z = 3$$

$$\Rightarrow z^2 - 2z - 3 = 0$$

$$\Rightarrow (z - 3)(z + 1) = 0$$

$$\Rightarrow z = 3 \text{ or } z = -1$$

$$S = \{3, -1\}$$

(d) $\frac{2}{x + 3} - \frac{1}{x - 3} = \frac{1}{x^2 - 9}$

$$\Rightarrow \frac{2(x - 3) - 1(x + 3)}{(x + 3)(x - 3)} = \frac{1}{(x + 3)(x - 3)}$$

$$\Rightarrow 2(x - 3) - 1(x + 3) = 1$$

$$\Rightarrow 2x - 6 - x - 3 = 1$$

$$\Rightarrow x = 10$$

$$S = \{10\}$$