This problem will be counted as problem five of exam 2 and will replace that problem on the exam. You are given this opportunity to redo the problem since I later realized that I neglected to give you a result that would have made the problem easier. Since you have more time available now to do a good job on the problem, I’m asking you to prove this result and then to use it in the proof that the square root of five is irrational.

**Lemma:** \( \forall n \in \mathbb{Z}, 5|(n^2) \rightarrow 5|n. \)

**Theorem:** \( \sqrt{5} \) is irrational.

To prove the Lemma use the method of contradiction along with the Quotient-Remainder Theorem. You will then need to break the proof into a number of cases based on the value of the remainder.

To prove the theorem, use contradiction and the lemma. The proof should now be fairly easy and obvious; it is similar to the proof that \( \sqrt{2} \) is irrational as given in the textbook.