Problem 9.26  The masses of the ladder and person are 18 kg and 90 kg, respectively. The center of mass of the 4-m ladder is at its midpoint. If $\alpha = 30^\circ$, what is the minimum coefficient of static friction between the ladder and the floor necessary for the person to climb to the top of the ladder? Neglect friction between the ladder and the wall.

Solution:  The weight of the ladder is $W = 18 \, g = 176.58 \, N$. The weight of the person is $P = 90 \, g = 882.9 \, N$. Let $h$ be the distance along the ladder of the person’s center of mass, and $L$ be the length of the ladder. The horizontal distance is. The sum of the moments about the top of the ladder:

$$\sum M = P(L \sin \alpha - x) - F_N L \sin \alpha + W \frac{L}{2} \sin \alpha + f L \cos \alpha = 0.$$  

From the sum of the forces, $\sum F_y = F_N - W - P = 0$, from which the normal force at the foot of the ladder is $F_N = W + P$. Substitute, solve for the friction force, and reduce algebraically:

$$f = \left( \frac{hL}{P} + \frac{1}{2}W \right) \tan \alpha.$$  

At the top of the ladder, $\frac{h}{L} = 1$, hence

$$f = \left( \frac{P + W}{2} \right) \tan \alpha = (883 + 88.3)(0.5774) = 560.72 \, N$$

At impending slip, $f = \mu_s F_N = \mu_s(P + W)$, from which

$$\mu_s = \frac{f}{P + W} = \frac{560.72}{1059.48} = 0.5292$$
**Problem 9.76** Suppose that in Problem 9.75, A weighs 800 lb and B weighs 400 lb. The coefficients of friction between all of the contacting surfaces are \( \mu_s = 0.15 \) and \( \mu_k = 0.12 \). Will B remain in place if the force \( F \) is removed?

**Solution:** The equilibrium conditions are: For the box A: Denote the normal force exerted by the wall by \( Q \), and the normal force exerted by the wedge by \( N \). The friction forces oppose motion.

\[
\begin{align*}
\sum F_y &= -W + N \cos \alpha + \mu_s N \sin \alpha + \mu_k Q = 0, \\
\sum F_x &= +\mu_s N \cos \alpha - N \sin \alpha + Q = 0.
\end{align*}
\]

For the wedge B. Denote the normal force on the lower surface by \( P \).

\[
\begin{align*}
\sum F_y &= -\mu_s N \cos \alpha - \mu_k \cos \alpha + P \cos \alpha + P \sin \alpha + N \sin \alpha = 0, \\
\sum F_x &= -N \cos \alpha + P \cos \alpha - \mu_k N \sin \alpha + \mu_k P \sin \alpha - W_B = 0.
\end{align*}
\]

(A comparison with the equilibrium conditions for Problem 9.75 will show that the friction forces are reversed, since for slippage the box A will move downward, and the wedge B to the right.) The strategy is to solve these equations for the required \( \mu_s \) to keep the wedge B in place when \( F = 0 \). The solution \( Q = 0, N = 787.8 \text{ lb}, P = 1181.8 \text{ lb} \) and \( \mu_s = 0.1763 \). Since the value of \( \mu_s \) required to hold the wedge in place is greater than the value given, the wedge will slip out.

**Problem 9.77** Between A and B, \( \mu_s = 0.20 \), and between B and C, \( \mu_s = 0.18 \). Between C and the wall, \( \mu_s = 0.30 \). The weights \( W_B = 20 \text{ lb} \) and \( W_C = 80 \text{ lb} \). What force \( F \) is required to start C moving upward?

**Solution:** The active contact surfaces are between the wall and C, between the wedge B and C, and between the wedge B and A. For the weight C: Denote the normal force exerted by the wall by \( Q \), and the normal force between B and C by \( N \). Denote the several coefficients of static friction by subscripts. The equilibrium conditions are:

\[
\begin{align*}
\sum F_y &= -W_C + N - \mu_{CW} Q = 0, \\
\sum F_x &= -Q + \mu_{BC} N = 0.
\end{align*}
\]

For the wedge B: Denote the normal force between A and B by \( P \).

\[
\begin{align*}
\sum F_y &= -N + P \cos \alpha - \mu_{AB} P \sin \alpha - W_B = 0, \\
\sum F_x &= -P - \mu_{BC} N - \mu_{AB} P \cos \alpha - P \sin \alpha = 0.
\end{align*}
\]

These four equations in four unknowns are solved:

\[
\begin{align*}
Q &= 15.2 \text{ lb}, \\
N &= 84.6 \text{ lb}, \\
P &= 114.4 \text{ lb},
\end{align*}
\]

and \( F = 66.9 \text{ lb} \)
Problem 9.74  Suppose that between all contacting surfaces in Problem 9.73, \( \mu_s = 0.32 \) and \( \mu_k = 0.30 \). Neglect the weights of the \( 5^\circ \) wedges. If a force \( F = 800 \text{ N} \) is required to move \( A \) to the right at a constant rate, what is the mass of \( A \)?

Solution: The free body diagrams of the left wedge and the combined right wedge and crate are as shown. The equilibrium equations are

Wedge:
\[
\sum F_x = N - P \cos 5^\circ + 0.3P \sin 5^\circ = 0,
\]
\[
\sum F_y = 0.3 \text{ N} + P \sin 5^\circ + 0.3P \cos 5^\circ - F = 0,
\]
Wedge and box:
\[
\sum F_x = P \cos 5^\circ - 0.3P \sin 5^\circ - 0.3Q = 0,
\]
\[
\sum F_y = Q - P \sin 5^\circ - 0.3P \cos 5^\circ - 9.81 \text{ m} = 0.
\]
Solving them, we obtain

\[
P = 1180 \text{ N},
\]
\[
N = 1150 \text{ N},
\]
\[
Q = 3820 \text{ N},
\]
and \( m = 343 \text{ kg} \).

Problem 9.75  The box \( A \) has a mass of 80 kg, and the wedge \( B \) has a mass of 40 kg. Between all contacting surfaces, \( \mu_s = 0.15 \) and \( \mu_k = 0.12 \). What force \( F \) is required to raise \( A \) at a constant rate?

Solution: From the free-body diagrams shown, the equilibrium equations are

Box \( A \):
\[
Q - N \sin 10^\circ - \mu_k N \cos 10^\circ = 0,
\]
\[
N \cos 10^\circ - \mu_k N \sin 10^\circ - \mu_k Q - W = 0.
\]
Wedge \( B \):
\[
P \sin 10^\circ + \mu_k P \cos 10^\circ + N \sin 10^\circ + \mu_k N \cos 10^\circ - F = 0
\]
\[
P \cos 10^\circ - \mu_k P \sin 10^\circ - N \cos 10^\circ + \mu_k N \sin 10^\circ - W_w = 0.
\]
Solving with
\[
W = (80)(9.81) \text{ N},
\]
\[
W_w = (40)(9.81) \text{ N},
\]
and \( \mu_k = 0.12 \),
we obtain
\[
N = 845 \text{ N},
\]
\[
Q = 247 \text{ N},
\]
\[
P = 1252 \text{ N},
\]
and \( F = 612 \text{ N} \).
Problem 9.29  The disk weighs 50 lb. Neglect the weight of the bar. The coefficients of friction between the disk and the floor are \( \mu_s = 0.6 \) and \( \mu_k = 0.4 \).

(a) What is the largest couple \( M \) that can be applied to the stationary disk without causing it to start rotating?
(b) What couple \( M \) is necessary to rotate the disk at a constant rate?

Solution:  The normal force at the point of contact is found from the sum of moments about the pin support.

\[
\sum M = -8(100) - 20(50) + 20F_N = 0,
\]
from which \( F_N = 90 \) lb. The friction force is \( f = \mu_s F_N \). The moment exerted by the friction force is

\[
M_F = \mu_s RF_N = 0.6(5)(90) = 270 \text{ in lb}
\]
This is the moment to be overcome at impending slip.

(b) The moment required to rotate the disk at a constant rate is

\[
M_K = \mu_k RF_N = 0.4(5)(90) = 180 \text{ in lb}
\]

Problem 9.30  The cylinder has weight \( W \). The coefficient of static friction between the cylinder and the floor and between the cylinder and the wall is \( \mu_s \). What is the largest couple \( M \) that can be applied to the stationary cylinder without causing it to rotate?

Solution:  Assume impending slip. The force opposing rotation is the sum of the friction force at the wall and at the floor. Denote the normal force at the wall by \( F_{NW} \) and the normal force on the floor by \( F_{NF} \). From the sum of forces:

\[
\sum F_y = \mu_s F_{NW} + F_{NF} - W = 0,
\]
and

\[
\sum F_x = F_{NW} - \mu_s F_{NF} = 0.
\]
Solve these two simultaneous equations to obtain:

\[
F_{NF} = \frac{W}{1 + \mu_s^2},
\]
and

\[
F_{NW} = \frac{\mu_s W}{1 + \mu_s^2}.
\]
The sum of moments about the center of the cylinder is

\[
\sum M_C = M_{app} - \mu_s RF_{NW} - \mu_s RF_{NF} = 0.
\]
Substitute and solve:

\[
M_{app} = \mu_s RW \left( \frac{1 + \mu_s}{1 + \mu_s^2} \right).
\]
At impending slip, this is the maximum moment that can be applied to the cylinder.
**Problem 9.131** The 20-kg box \(A\) is held in equilibrium on the inclined surface by the force \(T\) acting on the rope wrapped over the fixed cylinder. The coefficient of static friction between the box and the inclined surface is 0.1. The coefficient of static friction between the rope and the cylinder is 0.05. Determine the largest value of \(T\) that will not cause the box to slip up the inclined surface.

**Solution:** Assuming that slip of the box up the surface impends. The free body diagrams of the box and rope around the cylinder are as shown.

From the equilibrium equations

\[
\sum F_x = T_A \cos 45^\circ - N \sin 20^\circ - 0.1 \ N \cos 20^\circ = 0,
\]

\[
\sum F_y = T_A \sin 45^\circ + N \cos 20^\circ - 0.1 \ N \sin 20^\circ - (20)(9.81) = 0
\]

we obtain \(T_A = 90.2 \ \text{N}\). Equation (9.17) is \(T = T_A \mu_s \beta\) where \(\mu_s = 0.05\) and \(\beta = (\pi/180)(135)\) rad. Solving for \(T\) we obtain \(T = 101 \ \text{N}\).

**Problem 9.132** In Problem 9.131, determine the smallest value of \(T\) necessary to hold the box in equilibrium on the inclined surface.

**Solution:** In this case, we assume that slip of the box down the surface impends. This requires reversing the direction of the friction force in the free body diagram of Problem 9.131. The friction now acts up the surface and the friction on the drum is reversed. See the free body diagrams. From the equilibrium equations,

\[
\sum F_x = T_A \cos(45^\circ) - N \sin(20^\circ) + 0.1 \ N \cos(20^\circ) = 0,
\]

and \(\sum F_y = T_A \sin(45^\circ) + N \cos(20^\circ) + 0.1 \ N \sin(20^\circ) - (20)(9.81) = 0\).

Solving, we obtain \(T_A = 56.3 \ \text{N}\). We can now use this to find the force \(T\) that must be applied to the rope to keep the box from slipping down the plane. Eq. (9.17) is \(T = T_A \mu_s \beta\), where \(\mu_s = 0.05\) and \(\beta = (\pi/180)(135)\) rad. Solving for \(T\), we obtain \(T = 50.1 \ \text{N}\).
Problem 9.136  The spring exerts a 320-N force on the left pulley. The coefficient of static friction between the flat belt and the pulleys is $\mu_s = 0.5$. The right pulley cannot rotate. What is the largest couple $M$ that can be exerted on the left pulley without causing the belt to slip?

Solution:  The angle of the belt relative to the horizontal is

$$\alpha = \sin^{-1}\left(\frac{100 - 40}{260}\right) = 0.2329 \text{ radians.}$$

For the right pulley the angle of contact is $\beta_{right} = \pi - 2\alpha = 2.676 \text{ radians.}$ The sum of the horizontal components of the tensions equals the force exerted by the spring:

$$F = (T_{upper} + T_{lower}) \cos \alpha = 320 \text{ N.}$$

Since the angle of contact is less on the right pulley, it should slip there first. At impending slip, the tensions are related by

$$T_{upper} = T_{lower} e^{\mu_s \beta_{right}} = 3.811 T_{lower}.$$  

Substitute and solve:

$$T_{lower}(1 + e^{0.5(2.676)}) = \frac{320}{\cos \alpha},$$

from which

$$T_{lower} = 68.34 \text{ N,}$$

and $T_{upper} = 260.48 \text{ N.}$

The moment applied to the wheel on the right is

$$M_{applied} = R(T_{upper} - T_{lower}) = 0.1(192.16) = 19.22 \text{ N-m}$$

Problem 9.137  Suppose that the belt in Problem 9.136 is a V-belt with angle $\gamma = 30^\circ$. What is the largest couple $M$ that can be exerted on the left pulley without causing the belt to slip?

Solution:  Use the results of the solution to Problem 9.136, as applicable. The angle of contact with the right pulley is $\beta_{right} = 2.676 \text{ radians.}$ The sum of the horizontal components of the tensions equals the force exerted by the spring:

$$F = (T_{upper} + T_{lower}) \cos \alpha = 320 \text{ N.}$$

Since the angle of contact is less on the right pulley, it will slip there. At impending slip, the tensions are related by $T_{upper} = T_{lower} e^{\Psi}$, where for brevity

$$\Psi = \frac{\mu_s \beta_{right}}{\sin \left(\frac{\gamma}{2}\right)} = 5.169$$

has been substituted. Substitute and solve:

$$T_{lower}(1 + e^\Psi) = \frac{320}{\cos \alpha},$$

from which

$$T_{lower} = 1.86 \text{ N,}$$

and $T_{upper} = 327.01 \text{ N.}$

The moment applied to the wheel on the right is

$$M_{applied} = R(T_{upper} - T_{lower}) = 0.1(325.16) = 32.52 \text{ N-m}$$