Chapter 8 – Friction

(Dry)

- Coeffs. μ_s – no motion; μ_k – w/ motion "relative"
- opposite direction of motion

$\Sigma F_x = ma$

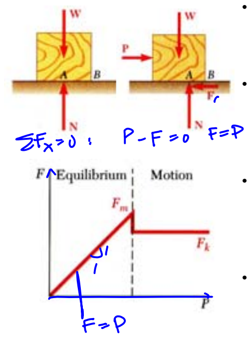
mg, F_f, N, $\vec{a}$, moves to the right

- tangential

$F_s \leq \mu_s N$ ← normal

$F_k = \mu_k N$

The Laws of Dry Friction



$\Sigma F_x = 0: \quad P - F = 0 \quad F = P$

- Block of weight *W* placed on horizontal surface. Forces acting on block are its weight and reaction of surface *N*.
- Small horizontal force *P* applied to block. For block to remain stationary, in equilibrium, a horizontal component *F* of the surface reaction is required. *F* is a *static-friction force.*
- As *P* increases, the static-friction force *F* increases as well until it reaches a maximum value F_m.

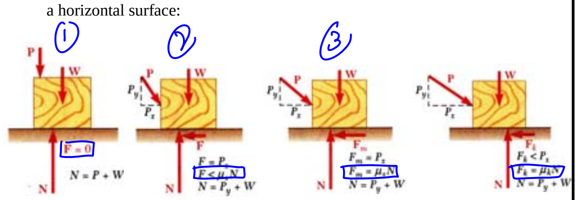
$\boxed{F_m = \mu_s N}$ ← impending motion

- Further increase in P causes the block to begin to move as *F* drops to a smaller *kinetic-friction force* F_k.

$\boxed{F_k = \mu_k N}$ $\quad \mu_s > \mu_k$

The Laws of Dry Friction

- Four situations can occur when a rigid body is in contact with a horizontal surface:



- No friction, $(P_x = 0)$
- No motion, $(P_x < F_m)$
- Motion impending, $(P_x = F_m)$
- Motion, $(P_x > F_m)$

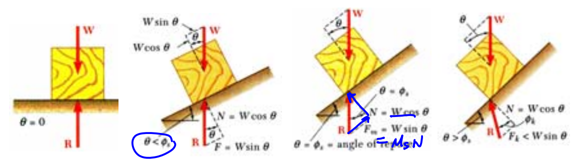
No motion: $F \leq \mu_s N$

motion: $F = \mu_k N$

Angles of Friction

- Consider block of weight *W* resting on board with variable inclination angle θ.

$\phi_s = \tan^{-1}(\mu_s)$



- No friction
- No motion — $F < \mu_s N$
- Motion impending — $F = \mu_s N$
- Motion — $F_k = \mu_k N$

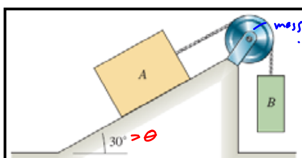
R, N, $F_m = \mu_s N$

$\tan\phi = \frac{F_m}{N} = \frac{\mu_s N}{N}$

$\tan\phi_s = \mu_s$

$\phi_s = \tan^{-1}(\mu_s)$

Example 1



Example 1. Determine the range of values of mass of B needed to keep the 100-kg block A in equilibrium. The coefficients of friction are: μ_s=0.3, μ_k=0.2.

$30° > \theta$

Minimum B needed?

$\phi_s = \tan^{-1}(\mu_s)$

$= \tan^{-1}(0.3)$

$= 16.7°$

$\theta > \phi_s$: need B.

Find min m_B:

A's motion is impending

FBD of A: $100g$, T, $F = 0.3N$, N

$\Sigma F_y' = 0 \Rightarrow N = 100g\cos 30°$

$\Sigma F_x' = 0 \Rightarrow 0.3N - 100g\sin 30° + T = 0$

$T = 100g\sin 30° - 0.3(100g\cos 30°)$

$= 50g - 30\cos 30° g$

$T = 235.6$ N

$m_B = \frac{235.6}{g} = 24.01$ kg MIN

Max m_B:

Block A

100g, T, $F = 0.3N$, N

$\Sigma F_y' = 0 \quad N = 100g\cos 30°$

$\Sigma F_x' = 0 \quad -0.3N - 100g\sin 30° + T = 0$

$T = 50g + 30g\cos 30°$

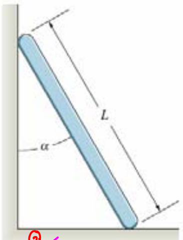
$= 745.4$ N

FBD of B

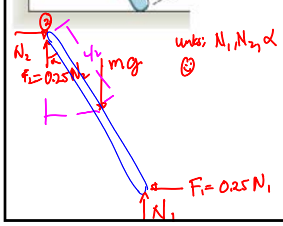
$m_B g = T$

$m_B = \frac{745.4}{g} = 75.90$ kg

$\boxed{24.01\text{ kg} \leq m_B \leq 75.90\text{ kg}}$



Example 2. A uniform rod of length L and mass m is leaned against a vertical wall. If the coefficient of static friction between all surfaces in contact is 0.25, find the maximum angle α for equilibrium.



unks: N_1, N_2, α ☺

Case: impending

$\Sigma F_x = 0$: $N_2 = 0.25 N_1$

$\Sigma F_y = 0$: $0.25 N_2 - mg + N_1 = 0$

$(0.25)^2 N_1 - mg + N_1 = 0$

$\boxed{N_1 = \frac{mg}{1+\mu_s^2}}$

$\Sigma M_B = 0$

$-mg\frac{L}{2}\sin\alpha + N_1 L \sin\alpha - \mu_s N_1 L \cos\alpha = 0$

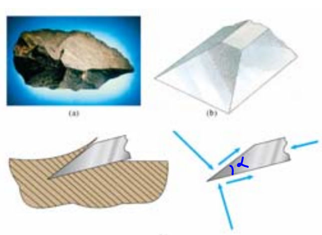
$-\frac{mg}{2}\sin\alpha + \frac{mg}{1+\mu_s^2}\sin\alpha - \mu_s\frac{mg}{1+\mu_s^2}\cos\alpha = 0$

$\sin\alpha\left(\frac{1}{1+\mu_s^2} - \frac{1}{2}\right) = \frac{\mu_s}{1+\mu_s^2}\cos\alpha$ $\boxed{\alpha = 28°}$

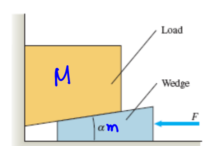
$\tan\alpha = \frac{\mu_s}{1+\mu_s^2} \div \frac{1-\mu_s^2}{2(1+\mu_s^2)} =$

Wedge

- A bifacial tool with faces set at a small acute angle.

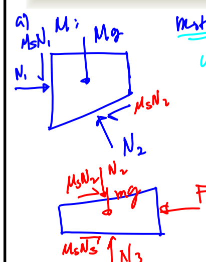


- Faces of a wedge can exert large lateral forces.



Find minimum force F needed to:
a) Raise block
b) Keep system in equilibrium

M_b $m \rightarrow$



motion of FBD relative to the external object.

unks: N_1 & N_2 ☺

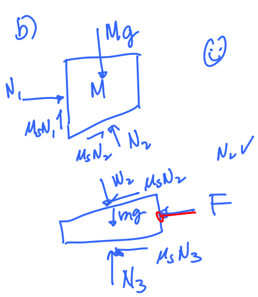
$\Sigma F_x = 0 \Rightarrow N_1, N_2$

$\Sigma F_y = 0 \Rightarrow N_1, N_2$ solve N_1, N_2

unks: N_2, N_3, F ☺

$\Sigma F_y = 0 \Rightarrow N_3$

$\Sigma F_x = 0 \Rightarrow F$

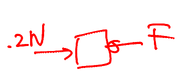


☺ N_1, N_2

N_2 ✓ ☺

$F > 0$

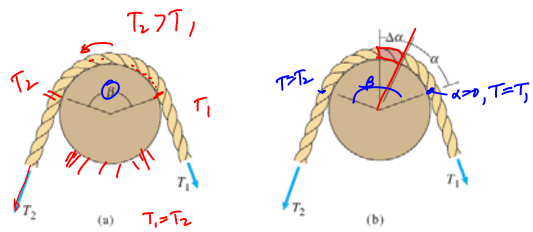
$F < 0$



$\Sigma F_x = 0$

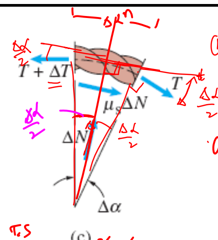
$F = .2N$

Belt Friction



Slowly increase T_2 until belt is about to slide. impending

Take a small section $\Delta\alpha$ of the belt (α)



EQUIL:

① $\Sigma F_n = 0$; $\Delta N - T\sin\left(\frac{\Delta\alpha}{2}\right) - (T + \Delta T)\sin\left(\frac{\Delta\alpha}{2}\right) = 0$

$\Sigma F_t = 0$

$\mu_s \Delta N + T\cos\left(\frac{\Delta\alpha}{2}\right) - (\cancel{T} + \Delta T)\cos\left(\frac{\Delta\alpha}{2}\right) = 0$

Approximations: $\Delta\alpha$ is small, $\frac{\Delta\alpha}{2}$ is small

$\boxed{\sin\left(\frac{\Delta\alpha}{2}\right) \approx \frac{\Delta\alpha}{2}}$ ($\Delta\alpha$ in radians)

$\boxed{\cos\left(\frac{\Delta\alpha}{2}\right) \approx 1}$

T.S

$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} \cdots$

$\cos x = 1 - \frac{x^2}{2!} + \cdots$

$\Delta N - 2T\left(\frac{\Delta\alpha}{2}\right) - \Delta T \frac{\Delta\alpha}{2} = 0$

$\Delta N \approx T\Delta\alpha + \Delta T \frac{\Delta\alpha}{2}$

$\lim_{\Delta\alpha \to 0} \left(\frac{\Delta N}{\Delta\alpha} \approx T + \cancel{\frac{\Delta T}{2}}\right) \Rightarrow \boxed{\frac{dN}{d\alpha} = T} \rightarrow dN = T\,d\alpha$

② $\mu_s \Delta N - \Delta T(1) \approx 0$

$\boxed{dT = \mu_s\, dN}$

$\boxed{dT = \mu_s T\, d\alpha}$ separable

$\int_{T_1}^{T_2} \frac{dT}{T} = \int_0^\beta \mu_s\, d\alpha$

$\ln T \Big|_{T_1}^{T_2} = \mu_s \alpha \Big|_0^\beta$

$e^{\ln \frac{T_2}{T_1}} = e^{\mu_s \beta}$ $\boxed{T_2 = T_1 e^{\mu_s \beta}}$

Motion Impending: *

$\boxed{T_L = T_s e^{\mu_s \beta}}$

T_s = small tension

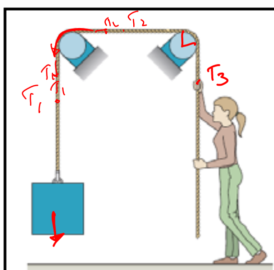
T_L = larger "

β = contact angle in rad.

w/ Sliding:

$\boxed{T_L = T_s e^{\mu_k \beta}}$

Example.



Given the mass of block: $\mu_s = 0.2$ $\mu_k = 0.1$

a) Find minimum force needed to keep block in equil.

b) Find minimum force needed to lift the block.

a) Impending motion of block ↓

rope motion:

$T_1 > T_2 > T_3$

$\boxed{T_L = T_s e^{\mu_s \beta}}$

$T_1 = T_2 e^{0.2(\frac{\pi}{2})}$

$mg = T_2 e^{0.2(\frac{\pi}{2})}$

$\boxed{T_2 = mg\, e^{-0.2(\frac{\pi}{2})}}$

$T_2 > T_3$

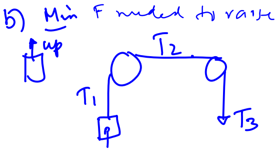
$T_2 = T_3 e^{\mu_s \beta}$

$mg\, e^{-0.2(\frac{\pi}{2})} = T_3 e^{0.2(\frac{\pi}{2})}$

$\boxed{T_3 = mg\, e^{-0.2\pi}}$

$= 0.53\, mg$

b) Min F needed to raise block : impending



$T_2 > T_1$; $T_3 > T_2$

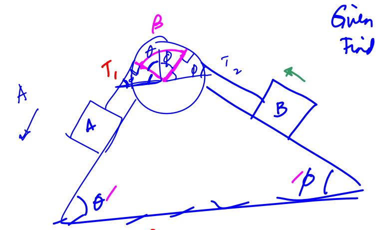
$T_L = T_s e^{\mu_s \beta}$

$T_2 = mg\, e^{0.2 \frac{\pi}{2}}$

$T_3 = T_2 e^{\mu_s \beta}$

$\boxed{T_3 = mg\, e^{0.2\pi}}$

$T_3 = 1.87\, mg$



Given m_A; μ_s
Find. min mass of B needed to keep A in equil.

$T_2 < T_1$

$T_1 = T_2 e^{\mu_s \beta}$

$\beta = \theta + \phi$

$T_2 = T_1 e^{-\mu_s(\theta+\phi)}$

FBD of A

$\Sigma F_{y'} = 0$

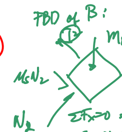
$N_1 = m_A g \cos\theta$

$\Sigma F_{x'} = 0$

$T_1 = m_A g \sin\theta - \mu_s m_A g \cos\theta$

$T_1 = m_A g (\sin\theta - \mu_s \cos\theta)$

FBD of B:



$\Sigma F_x = 0 \Rightarrow N_2$

$\Sigma F_y = 0 \Rightarrow \underline{m_B}$