69. The number of low-carbohydrate (low-carb) ice cream products in the years 2000, 2001, 2002, 2003, and 2004 is 0, 2, 9, 19, and 62, respectively (Source: *Productscan Online*). Let n be the number of low-carb ice cream products.

a. Use points on a number line to describe the given values of n. Find the average of the values and indicate it on the number line.

b. Did the number of low-carb ice cream products increase, decrease, stay approximately constant, or none of these between 2000 and 2004, inclusive? Explain.

c. Did the annual *increases* in the number of low-carb ice cream products increase, decrease, stay approximately constant, or none of these between 2000 and 2004, inclusive? Explain.

70. The sales (in millions) of digital cameras in the years 2000, 2001, 2002, 2003, and 2004 are 4.5, 7.0, 9.4, 13.0, and 18.2, respectively (Source: *Photo Marketing Association International*). Let s be the digital camera sales (in millions) in a year.

a. Use points on a number line to describe the given values of s. Find the average of the values and indicate it on the number line.

b. Did the annual digital camera sales increase, decrease, stay approximately constant, or none of these from 2000 to 2004, inclusive? Explain.

c. Did the *increases* in the annual digital camera sales increase, decrease, stay approximately constant, or none of these from 2000 to 2004, inclusive? Explain.

71. The number of complaints (in thousands) of identity theft to the Federal Trade Commission (FTC) in the years 2001, 2002, 2003, and 2004 is 86, 162, 215, and 247, respectively (Source: *FTC*). Let n be the number of complaints (in thousands) in a year.

a. Use points on a number line to describe the given values of n. Find the average of the values and indicate it on the number line.

b. Did the number of complaints per year increase, decrease, stay approximately constant, or none of these between 2001 and 2004, inclusive? Explain.

c. Did the *increases* in the number of complaints per year increase, decrease, stay approximately constant, or none of these between 2001 and 2004, inclusive? Explain.

72. The annual total profits (in billions of dollars) of credit card companies in the years 1999, 2000, 2001, 2002, 2003, and 2004 are 14, 20, 24, 27, 30, and 32, respectively (Sources: *CardWeb.com; CardData*). Let p be the total profit (in billions of dollars) of credit card companies in a year.

a. Use points on a number line to describe the given values of p. Find the average of the values and indicate it on the number line.

b. Did the annual total profit increase, decrease, stay approximately constant, or none of these from 1999 to 2004, inclusive? Explain.

c. Did the *increases* in the annual total profit increase, decrease, stay approximately constant, or none of these from 1999 to 2004, inclusive? Explain.

73. Let T be the temperature in degrees Fahrenheit.

a. What value of T represents the temperature that is 5°F below zero?

b. A student says that T represents only positive numbers and zero, because there is no negative sign. Is the student correct? Explain.

74. A student says that the integers between 2 and 5 are the numbers 2, 3, 4, and 5. Is the student correct? Explain.

75. a. Find the average of each pair of numbers. Then plot the two given numbers and their average on a number line.

i. 7, 9 **ii.** 1, 5 **iii.** 2, 8

b. What patterns do you notice in your work in part (a)?

c. How many real numbers are there between 0 and 1? Explain. [**Hint:** Use the concept of average to help you show that you can keep finding more numbers.]

76. We can describe how far apart two numbers are on the number line. For example, the numbers 3 and 7 are 4 units apart. How far apart are two consecutive integers on the number line? How far apart are two consecutive even integers? How far apart are two consecutive odd integers?

77. List the various types of numbers discussed in this section and describe the meanings of each type. (See page 9 for guidelines on writing a good response.)

78. Describe how to graph a negative quantity. (See page 9 for guidelines on writing a good response.)

1.2 SCATTERGRAMS

Objectives

- Know the meaning of *ordered pair, coordinate,* and *coordinate system.*
- Create scattergrams.
- Know the meaning of *independent variable* and *dependent variable.*
- Read bar graphs.
- Plot points on a coordinate system.

In Section 1.1, we used a single number line to describe values of a quantity. In this section, we will use a pair of number lines to describe two quantities that are related. For instance, in Example 3 we will compare the average price of a Super Bowl ticket with the Super Bowl number.

When One Number Line Is Not Enough

A number line is convenient for displaying values of a variable. However, there are limitations to using a *single* number line. For example, suppose a student earns the following points (listed in chronological order) from taking five quizzes: 7, 6, 9, 8, 9. We let q stand for the number of points earned by the student on a quiz. We use a number line to graph the scores in Fig. 15.

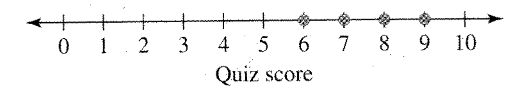

Figure 15 Graphing the scores

There are two limitations with using one number line to graph these scores. First, the graph does not show that there were two scores of 9 points each. Second, the graph does not show which was the first score, the second score, and so on.

The Coordinate System

There is a way to address both limitations. To begin, we let n be the quiz number. For the first quiz, $n = 1$. For the second quiz, $n = 2$, and so on. Next, we organize the values of the variables n and q (the quiz score) in Table 2.

Table 2 Values of n and q

n	q
1	7
2	6
3	9
4	8
5	9

The "1" and "7" in the first row of Table 2 indicate that when $n = 1$, $q = 7$. This means that the student's score on the first quiz was 7 points. If we agree to write the quiz number first and the quiz score second, we can use the ordered pair $(1, 7)$ to mean that when $n = 1$, $q = 7$. An **ordered pair** is a pair of numbers (written in parentheses and separated by a comma) for which the order of the numbers is meaningful. We call each of the numbers in an ordered pair a **coordinate.** For $(1, 7)$ in this situation, we call 1 the *n-coordinate* and 7 the *q-coordinate*.

The ordered pair $(2, 6)$ indicates that when $n = 2$, $q = 6$. This means that the student's score on the second quiz was 6 points, which agrees with the second row of Table 2.

We call pairs of numbers such as $(3, 9)$ ordered pairs, because the order in which the numbers appear matters: The ordered pair $(3, 9)$ means that the student's score on the third quiz was 9 points, whereas the ordered pair $(9, 3)$ means that the student's score on the ninth quiz was 3 points.

We graph the ordered pairs by using *two* number lines, which are called **axes** (singular: **axis**). To start, we draw a horizontal number line called the n-axis and a vertical number line called the q-axis (see Fig. 16). We refer to such a pair of axes as a **coordinate system.** The **origin** is the intersection point of the axes. The axes divide the coordinate system into four regions called **quadrants,** which we call Quadrants I, II, III, and IV. The quadrants do not include the axes.

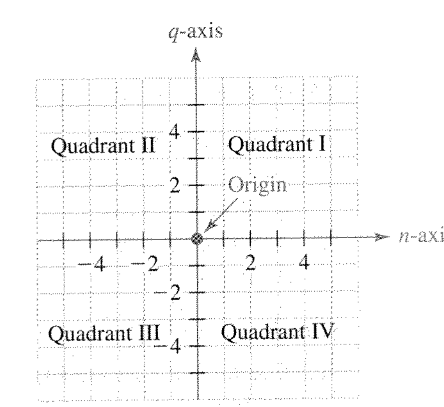

Figure 16 Coordinate system

Scattergrams

Next, we plot the ordered pair (3, 9) shown in the third row of Table 2. To do so, we start at the origin, look 3 units to the right and 9 units up, and then draw a dot (see Fig. 17). In Fig. 18 we plot all the ordered pairs listed in Table 2.

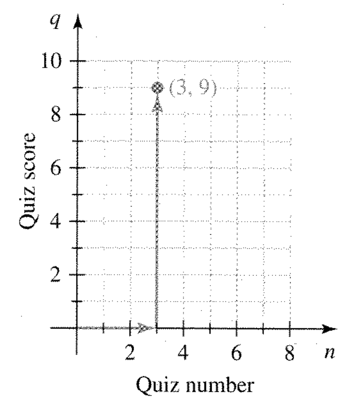

Figure 17 Plot (3, 9)

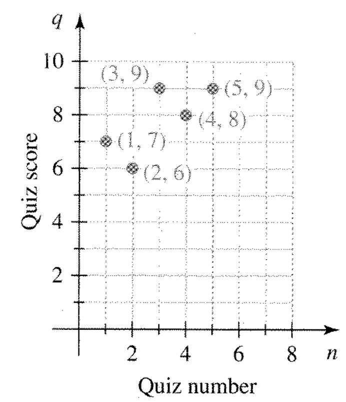

Figure 18 Plot the ordered pairs from Table 2

Note that we have addressed both of the limitations of using just one number line. The coordinate system in Fig. 18 shows that there are two scores of 9, and it also shows which score is from the first quiz, the second quiz, and so on.

As we look at the plotted points in Fig. 18 from left to right, the points, in general, go upward. This means that the quiz scores, in general, are increasing.

A graph of plotted ordered pairs, such as the graph in Fig. 18, is called a **scattergram.**

Independent and Dependent Variables

The student's score q on a quiz depends on the quiz number n. For example, if a certain quiz is more difficult than the others, the student may earn a lower score on that quiz than on the other quizzes. Because q depends on n, we call q the *dependent variable*.

The quiz number does not depend on the student's score, however—n is independent of q. We call n the *independent variable*.

DEFINITION Identifying independent and dependent variables

Assume that an authentic situation can be described by using the variables t and p, and assume that p depends on t. Then

- We call t the **independent variable.**
- We call p the **dependent variable.**

Example 1 Identifying Independent and Dependent Variables

For each situation, identify the independent variable and the dependent variable.

1. Let L be the loudness (in decibels) of the sound produced by a foghorn that is d miles away from you.
2. A car is traveling at speed s (in mph) on a dry asphalt road, and the brakes are suddenly applied. Let d be the stopping distance (in feet).

Solution

1. The farther you are from the foghorn, the softer the sound you will hear. So, the loudness L depends on the distance d. Thus, L is the dependent variable and d is the independent variable. (Notice that the distance does *not* depend on the loudness.)

2. The greater the traveling speed, the greater the stopping distance will be. So, the stopping distance d depends on the traveling speed s. Thus, d is the dependent variable and s is the independent variable. (Notice that the traveling speed does *not* depend on the stopping distance.) ■

For an ordered pair (a, b), we write the value of the independent variable in the first (left) position and the value of the dependent variable in the second (right) position. Note that we did just that for the quiz score application. That is, we listed values of the independent variable n in the first position and values of the dependent variable q in the second position. For example, the ordered pair (4, 8) means that when $n = 4$, $q = 8$. In other words, the student's score on the fourth quiz is 8 points.

Example 2 Determining the Meaning of an Ordered Pair

1. Let n be the total number of Quiznos Sub® restaurants at t years since 2000. What does the ordered pair (4, 3500) mean in this situation?
2. Let p be a runner's pulse rate (in beats per minute) when his speed is s miles per hour. What does the ordered pair (10, 160) mean in this situation?

Solution

1. The total number of restaurants depends on the year. So, n is the dependent variable and t is the independent variable. The ordered pair (4, 3500) means that $t = 4$ and $n = 3500$. There were 3500 restaurants in $2000 + 4 = 2004$.
2. The runner's pulse rate depends on his speed. So, p is the dependent variable and s is the independent variable. The ordered pair (10, 160) means that $s = 10$ and $p = 160$. When the runner's speed is 10 miles per hour, his pulse rate is 160 beats per minute. ■

Table 3 Values of n and q

n	q
1	7
2	6
3	9
4	8
5	9

For tables of ordered pairs, we list the values of the independent variable in the first (left) column and the values of the dependent variable in the second (right) column. For example, in Table 3 the values of n are in the first column and the values of q are in the second column.

For coordinate systems, we describe the values of the independent variable with the horizontal axis and the values of the dependent variable with the vertical axis. For example, in Fig. 18 the horizontal axis is the n-axis and the vertical axis is the q-axis.

Columns of Tables and Axes of Coordinate Systems

Assume that an authentic situation can be described by using two variables. Then

- For tables, the values of the independent variable are listed in the first column and the values of the dependent variable are listed in the second column (see Table 4).
- For coordinate systems, the values of the independent variable are described by the horizontal axis and the values of the dependent variable are described by the vertical axis (see Fig. 19).

Table 4 Position of the Variables

Independent Variable	Dependent Variable
*	*
*	*
*	*

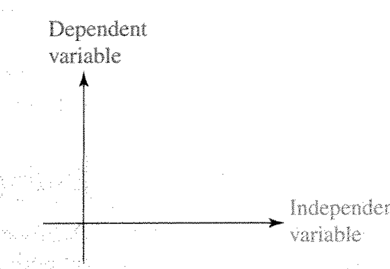

Figure 19 Positions of the variables

More about Scattergrams

Example 3 Creating a Scattergram

Table 5 Average Ticket Prices for the Super Bowl

Super Bowl Number	Average Ticket Price (dollars)
1 (I)	12
5 (V)	15
10 (X)	20
15 (XV)	40
20 (XX)	75
25 (XXV)	150
30 (XXX)	250
35 (XXXV)	325
40 (XL)	550

Source: *NFL.com*

The average ticket prices for the Super Bowl are shown in Table 5 for various years. Let p be the average ticket price (in dollars) and n be the Super Bowl number.

1. Draw a scattergram of the data.
2. For the Super Bowls described in Table 5, which Super Bowl had the highest average ticket price? What was that price?
3. Describe any patterns you see in the prices.

Solution

1. A scattergram of the data is shown in Fig. 20. It makes sense to think of p as the dependent variable, because the average ticket price depends on the Super Bowl number (and not the other way around). So, we let the vertical axis be the p-axis. Note that we write the variable names "n" and "p" and the units "Super Bowl number" and "Dollars" on the appropriate axes.

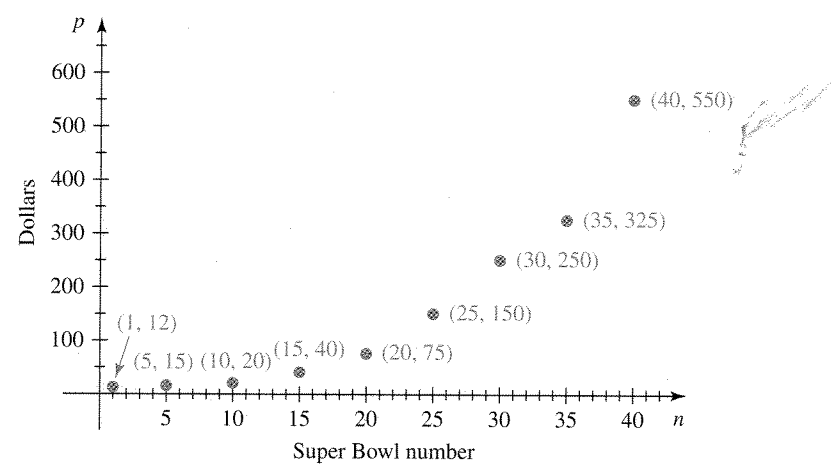

Figure 20 Scattergram of average Super Bowl ticket prices

Recall from Section 1.1 that when we write numbers on an axis, they should increase by a fixed amount and be equally spaced. Since the Super Bowl numbers are between 1 and 40, inclusive, we write the numbers 5, 10, 15, . . . , 40 equally spaced on the n-axis. Since the prices shown in Table 5 are between \$12 and \$550, inclusive, we write the numbers 100, 200, 300, 400, 500, and 600 on the p-axis.

The ordered pair (1, 12) indicates that the average ticket price was \$12 for Super Bowl I.

2. From Table 5 and the scattergram in Fig. 20, we see that the highest average ticket price was \$550 in Super Bowl XL.
3. From Table 5 and the scattergram in Fig. 20, we see that average ticket prices have increased. We also see that from Super Bowl I to Super Bowl XXX, inclusive, the *increases* in average ticket prices have increased. That is, as we look from left to right at the first seven points plotted on the coordinate system, we see that the vertical distance between adjacent points increases. ■

In Example 3, creating a scattergram helped us notice some patterns. Such observations will help us make predictions about authentic situations later in this course.

Example 4 Creating a Scattergram with Age Groups

A householder is the person in whose name a house, condominium, or apartment is owned or rented. The percentages of householders who own a home are listed in Table 6 for various age groups.

Table 6 Percentages of Householders Who Own a Home

Age Group (years)	Age Used to Represent Age Group (years)	Percent
15–24	19.5	18
25–34	29.5	46
35–44	39.5	66
45–54	49.5	75
55–64	59.5	80
65–74	69.5	81
75–84	79.5	77

Source: *U.S. Census Bureau*

Let p be the percentage of householders who own a home when they are at age a years.

1. Draw a scattergram of the data.
2. Describe any patterns in the percentages of householders who own a home.

Solution

1. A look at the first row of Table 6 suggests that we use $a = 19.5$ to represent the age group from 15 years to 24 years. The age 19.5 years is the average of the ages 15 years and 24 years. (Try it.) Likewise, we will use 29.5 to represent the age group from 25 years to 34 years and so on.

A scattergram of the data is shown in Fig. 21. It makes sense to think of p as the dependent variable, because the percentage of householders depends on the age group (and not the other way around). So, we let the vertical axis be the p-axis. Note that we write the variable names "a" and "p" and the units "Age in years" and "Percent" on the appropriate axes.

The ordered pair (19.5, 18) indicates that, for the age group from 15 years to 24 years, 18% of householders own a home.

Since the ages used to represent age groups are between 19.5 years and 79.5 years, inclusive, we write the numbers 10, 20, 30, . . . , 80 equally spaced on the a-axis. Since the percents shown in Table 6 are between 18% and 81%, inclusive, we write the numbers 20, 40, 60, 80 and 100 equally spaced on the p-axis.

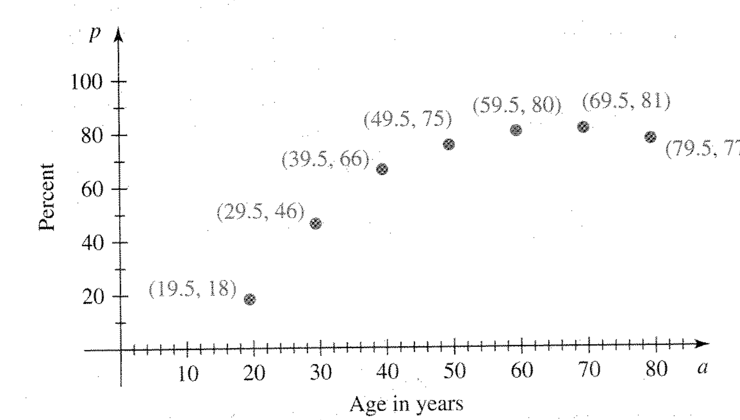

Figure 21 Scattergram of homeowner data

2. From Table 6 and the scattergram in Fig. 21, we see that until about age 70, the percentage of householders who own a home increases as their age increases. After about age 70, the percentage of householders who own a home decreases as their age increases. We also see that until about age 70, the increase in the percentage is decreasing: After every 10 years, the percentage of householders who own a home increases, but by less than it did in the previous 10 years. ■

In Example 5, we will define a variable to represent time.

Example 5 Defining a Variable for Time

Let t be the number of years since 1990. Find the values of t that represent the years 1990, 1996, 1999, 2000, and 2003.

Solution

We can represent 1990 by $t = 0$, because 1990 is 0 years after 1990. We can represent 1996 by $t = 6$, because 1996 is 6 years after 1990. We list the value of t for each of the years 1990, 1996, 1999, 2000, and 2003 in Table 7.

Table 7 Values of t

Year	Years since 1990 t	
1990	0	because $1990 - 1990 = 0$
1996	6	$1996 - 1990 = 6$
1999	9	$1999 - 1990 = 9$
2000	10	$2000 - 1990 = 10$
2003	13	$2003 - 1990 = 13$

The values of t in Table 7 are much smaller numbers than the years they represent. When working with authentic situations, we will often perform calculations that involve years. Using definitions similar to the one in Example 5 will enable us to perform those calculations with smaller numbers. It is also easier to label the axes of a coordinate system with smaller numbers.

Example 6 Creating a Scattergram with Zigzag Lines on an Axis

A *life expectancy* is a prediction of how long a person will live. Table 8 shows life expectancies at birth for Americans in various years.

Let L be the life expectancy at birth (in years) for an American born t years after 1980.

1. Create a scattergram of the data.
2. Describe any patterns in life expectancies of Americans.

Table 8 Life Expectancies

Year of Birth	Life Expectancy (years)
1980	73.7
1985	74.7
1990	75.4
1995	75.8
2000	76.9
2003	77.6

Source: *U.S. Census Bureau*

Solution

1. First, we list the values of t and L in Table 9. For example, $t = 0$ represents 1980, because 1980 is 0 years after 1980. Also, $t = 5$ represents 1985, because 1985 is 5 years after 1980.

   A scattergram of the data is shown in Fig. 22. Since L is the dependent variable, we let the vertical axis be the L-axis. We use zigzag lines on the L-axis to indicate that the part of the axis between 0 and about 71 is not displayed. This is done so that we can show a clear view of the data points without having to make the coordinate system exceedingly tall.

Table 9 Values of t and L

t (years since 1980)	L (years of life)
0	73.7
5	74.7
10	75.4
15	75.8
20	76.9
23	77.6

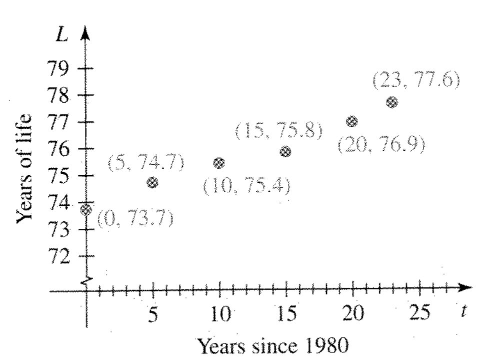

Figure 22 Life expectancy scattergram

2. From Table 9 and Fig. 22, we see that the life expectancy of Americans has been increasing fairly steadily since 1980. ■

Using zigzag lines on the L-axis in Fig. 22 helped us have a clear view of the data points. To see how poor a view we have without the zigzag lines, see Fig. 23.

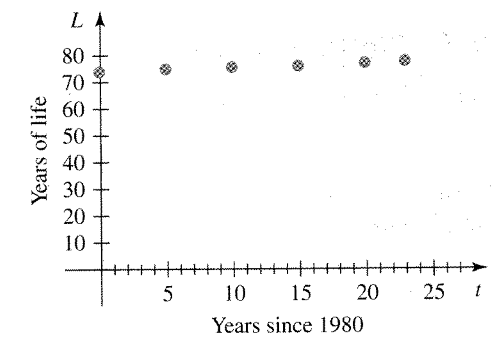

Figure 23 Poor view of life expectancy scattergram (Fig. 22), without zigzag lines

Without the zigzag lines, it is very difficult to plot the data points at the correct height. Also, in Section 1.3 we will begin to estimate coordinates of points by reading graphs, and it would be very difficult to do so accurately by using the graph in Fig. 23. In fact, it is hard to tell that the heights of the points increase from left to right.

Bar Graphs

A **bar graph** is a diagram with two axes that we can use to compare measurements of two or more items (see Fig. 24). Along one axis we list the items, and along the other axis we mark tick marks, write numbers, and write the units of the measurements. We use a bar to indicate the measurement of each item.

Example 7 Reading a Bar Graph

The revenues (in millions of dollars) of some Broadway-based movie musicals are illustrated in the bar graph in Fig. 24 (Source: *Box Office Mojo*).

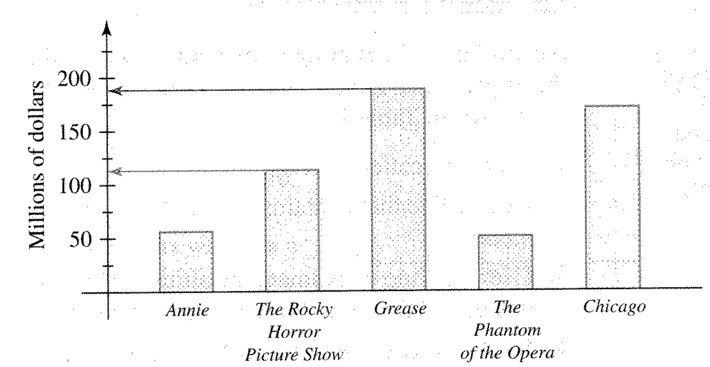

Figure 24 Bar graph of lifetime revenues of some Broadway-based movie musicals

1. Estimate the revenue of *The Rocky Horror Picture Show*.
2. Which Broadway-based movie musical listed in the bar graph has had the largest revenue? What is that revenue?

Solution

1. We look at the top of the bar for *The Rocky Horror Picture Show* and then look to the left at the vertical axis. It appears that the revenue is about \$113 million.
2. We identify the tallest bar, which is the one for *Grease*. We look at the top of the bar and then look to the left at the vertical axis. It appears that the revenue is about \$188 million. ■

Plotting Points on a Coordinate System

When we plot points that are not being used to describe authentic situations, we call the horizontal axis the *x-axis* and the vertical axis the *y-axis*. Then x is the independent variable and y is the dependent variable. The ordered pair $(6, 3)$ means that $x = 6$ and $y = 3$. So, the x-coordinate is 6 and the y-coordinate is 3.

Example 8 Plotting Points

Plot the points $(3, 4)$, $(-5, -3)$, $(-4, 2)$, and $(5, -4)$ on a coordinate system.

Solution

We plot the ordered pairs $(3, 4)$ and $(-5, -3)$ in Fig. 25, and we plot the ordered pairs $(-4, 2)$ and $(5, -4)$ in Fig. 26.

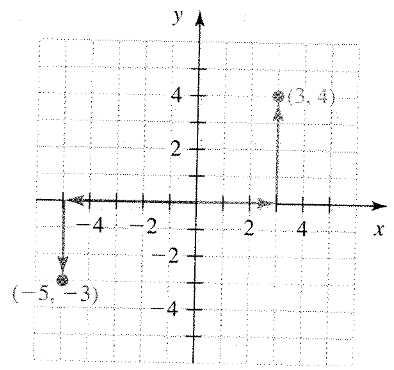

Figure 25 Plotting the ordered pairs $(3, 4)$ and $(-5, -3)$

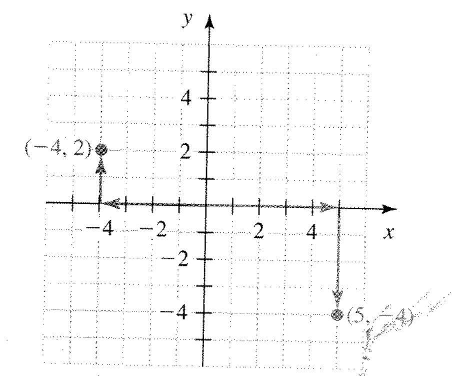

Figure 26 Plotting the ordered pairs $(-4, 2)$ and $(5, -4)$

group exploration

Looking ahead: Linear modeling

An airplane is beginning to descend. Let a be the altitude (in thousands of feet) of the airplane at t minutes since it began its descent. Some pairs of values of t and a are shown in Table 10.

Table 10 Altitudes of an Airplane

Time (minutes) t	Altitude (thousands of feet) a
0	24
5	20
10	16
15	12
20	8

1. Create a scattergram of the data.
2. Draw a line that contains the points in your scattergram. We call the line a *linear model*.
3. Use your line to estimate the altitude of the airplane 8 minutes after it began its descent. [**Hint:** On the line, locate the point whose t coordinate is 8.]
4. Use your line to estimate when the airplane reached an altitude of 10 thousand feet.
5. What was the altitude of the airplane when it began its descent?
6. Use your line to estimate when the airplane reached the ground.

TIPS FOR SUCCESS: Study Time

For each hour of class time, study for at least two hours outside class. If your math background is weak, you may need to spend more time studying.

One way to study is to do what you are doing now: Read the text. Class time is a great opportunity to be introduced to new concepts and to see how they fit together with previously learned ones. However, there is usually not enough time to address details as well as a textbook can. In this way, a textbook can serve as a supplement to what you learn in class.

HOMEWORK 1.2

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Plot the given points in a coordinate system.

1. $(5, 1)$ 2. $(2, 3)$ 3. $(4, -2)$ 4. $(3, -4)$
5. $(-5, 4)$ 6. $(-1, 3)$ 7. $(-3, -6)$ 8. $(-5, -2)$
9. $(0, 2)$ 10. $(0, -4)$ 11. $(-3, 0)$ 12. $(1, 0)$
13. $(2.5, -4.5)$ 14. $(-3.5, 1.5)$
15. $(-1.3, -3.9)$ 16. $(-2.4, -4.1)$
17. What is the x-coordinate of the ordered pair $(2, -4)$?
18. What is the y-coordinate of the ordered pair $(2, -4)$?

For Exercises 19–28, identify the independent variable and the dependent variable.

19. Let n be the number of hours that a student studies for a quiz, and let s be the student's score (in points) on the quiz.
20. Let t be the number of years a person has worked for a company, and let s be the person's salary (in dollars).
21. Let h be the height (in inches) of a girl, and let a be the age (in years) of the girl.
22. Let p be the percentage of colleges that would accept a student whose grade point average (GPA) is g points.
23. Let T be the tuition (in dollars) for enrolling in c credits (units or hours) of classes.
24. Let p be the percentage of men at age a years who have gray hair.
25. Let A be the floor area (in square feet) of a classroom, and let n be the number of students who can comfortably fit into the classroom.
26. A person cooks a potato in an oven for an hour and then removes the potato and allows it to cool. Let t be the number of minutes since the potato was removed from the oven, and let F be the temperature (in degrees Fahrenheit) of the potato.
27. Let t be the number of seconds after a baseball is hit upward, and let h be the baseball's height (in feet).
28. Let p be the percentage of people at age a years who own a computer.

For Exercises 29–36, describe what the given ordered pair represents.

29. Let n be the average number of magazine subscriptions sold per week by a telemarketer who works t hours per week. What does the ordered pair $(32, 43)$ mean in this situation?
30. Let c be the total cost (in dollars) of buying n pens. What does the ordered pair $(5, 10)$ mean in this situation?
31. Let p be the percentage of Americans at age A years who say they volunteer. What does the ordered pair $(21, 38)$ mean in this situation?
32. Let p be the percentage of Americans who have ever purchased a movie on pay-per-view at t years since 1995. What does the ordered pair $(6, 30)$ mean in this situation?
33. Let b be the amount of defense spending (in billions of dollars) at t years since 2000. What does the ordered pair $(2, 328)$ mean in this situation?
34. Let a be the total amount of money spent on ads (in billions of dollars) in the United States at t years since 2000. What does the ordered pair $(1, 106.6)$ mean in this situation?
35. Let p be the number of travelers (in millions) who booked trips online at t years since 2005. What does the ordered pair $(-2, 42)$ mean in this situation?
36. Let p be the percentage of Americans who have confidence in executives running major corporations at t years since 2005. What does the ordered pair $(-1, 12)$ mean in this situation?
37. Create a scattergram of the ordered pairs listed in Table 11.

Table 11 Some Ordered Pairs

x	y
2	5
7	9
11	10
14	9
16	5

38. Create a scattergram of the ordered pairs listed in Table 12.

Table 12 Some Ordered Pairs

x	y
−5	2
−4	3
−1	5
4	9
11	15

39. Find the coordinates of points A, B, C, D, E, and F shown in Fig. 27.

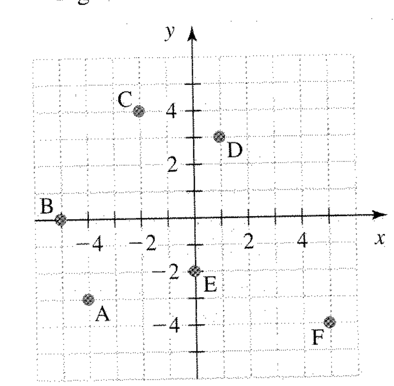

Figure 27 Exercise 39

40. Find the coordinates of points A, B, C, D, E, and F shown in Fig. 28.

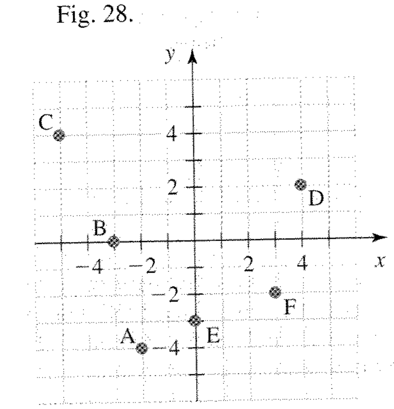

Figure 28 Exercise 40

41. The number of pages in each of the books in the Harry Potter series is listed in Table 13.

Table 13 Numbers of Pages in the Books of the Harry Potter Series

Book Number	Number of Pages
1	309
2	341
3	435
4	734
5	870
6	652

Let p be the number of pages and b be the corresponding book number.

a. Create a scattergram of the data.
b. Which book has the greatest number of pages?
c. From which book to the next did the number of pages increase the most? Explain how you can tell this by inspecting your scattergram.

42. The average life-spans of various denominations of bills are shown in Table 14. For example, the average life-span of a \$5 bill is 2 years before it is taken out of circulation due to wear and tear.

Table 14 Life-spans of Denominations of Bills

Value of Bill (dollars)	Life-span (years)
1	1.5
5	2
10	3
20	4
50	9

Source: *Federal Reserve System*

Let L be the life-span (in years) of a bill that is worth d dollars.

a. Create a scattergram of the data.
b. Explain why it makes sense that the average life-span of a \$50 bill is greater than the average life-span of a \$1 bill.
c. Each year, many more \$1 bills are printed than \$50 bills. Give at least two reasons that this makes sense.

43. The average times that mothers spend doing paid work per week are shown in Table 15 for various years.

Table 15 Average Time Mothers Spend Doing Paid Work

Year	Average Time (hours)
1965	9
1975	16
1985	21
1995	26
2003	22

Source: *Bureau of Labor Statistics*

a. Let a be the average time (in hours) that mothers spend doing paid work per week at t years since 1960. For example, $t = 0$ represents 1960, and $t = 5$ represents 1965. Create a scattergram of the data.
b. For which of the years shown in Table 15 was the average time that mothers spent doing paid work the greatest? What was that average time?
c. For which of the years shown in Table 15 was the average time that mothers spent doing paid work the least? What was that average time?

44. The numbers of books published about cats are shown in Table 16 for various years.

Table 16 Numbers of Books Published About Cats

Year	Number of Books
1999	138
2000	98
2001	92
2002	73
2003	73
2004	120

Source: *Books in Print, 2005*

a. Let n be the number of books published about cats in the year that is t years since 1990. For example, $t = 0$ represents 1990, and $t = 9$ represents 1999. Create a scattergram of the data.
b. In what year from 1999 to 2004 was there the greatest number of books published about cats? How many were published?
c. In what year from 1999 to 2004 was there the least number of books published about cats? How many were published?

45. The numbers of new products containing the artificial sweetener Splenda® are shown in Table 17 for various years.

Table 17 Numbers of New Products Containing Splenda

Year	Number of New Products Containing Splenda
2000	183
2001	261
2002	365
2003	561
2004	1330

Source: *Productscan Online*

Let n be the number of new products containing Splenda in the year that is t years since 2000.

a. Create a scattergram of the data.
b. Did the number of new products that contain Splenda increase, decrease, stay approximately constant, or none of these? Explain.
c. Did the annual *increase* in the number of new products that contain Splenda increase, decrease, stay approximately constant, or none of these? Explain.

46. The amounts of money that Americans spend annually on clothing for infants, toddlers, and preschoolers are shown in Table 18 for various years. Let s be the annual amount (in billions of dollars) spent by Americans on clothing for infants, toddlers, and preschoolers in the year that is t years since 2000.

a. Create a scattergram of the data.
b. Did the annual amount spent by Americans on clothing for infants, toddlers, and preschoolers increase, decrease, stay approximately constant, or none of these?

Table 18 Monies Spent on Clothing for Infants, Toddlers, and Preschoolers

Year	Money Spent (billions of dollars)
2000	13.2
2001	13.9
2002	14.7
2003	15.4
2004	16.1

Source: *Packaged Facts*

c. Did the *increase* in the annual amount spent by Americans on clothing for infants, toddlers, and preschoolers increase, decrease, stay approximately constant, or none of these?

47. The numbers of automobile accidents per 1000 licensed drivers are shown in Table 19 for various age groups.

Table 19 Automobile Accidents

Age Group (years)	Age Used to Represent Age Group (years)	Accident Rate (number of accidents per 1000 licensed drivers)
16	16	190.3
17	17	163.2
18	18	142.9
19	19	127.8
20–29	24.5	91.4
30–39	34.5	54.7
40–49	44.5	43.9
50–59	54.5	36.4
60–69	64.5	31.3
over 69	75	32.1

Source: *National Highway Traffic Safety Administration*

Let r be the automobile accident rate (number of accidents per 1000 licensed drivers) for licensed drivers at age a years.

a. Create a scattergram of the data.
b. Which age group shown in Table 19 has the lowest accident rate?
c. Which age group shown in Table 19 has the highest accident rate?
d. Between what two consecutive drivers' ages does there seem to be the greatest change in the accident rate? Explain why we can't be sure that this is true, because of the way the data are described in Table 19.
e. Many states put limits on teenage driving. For example, some states do not allow 16-year-old drivers to drive at night. Some states require parental supervision at all times. Why do you think that these regulations were adopted?

48. The percentages of Americans of various age groups who are ordering more takeout food than they did two years ago are shown in Table 20. Let p be the percentage of Americans at age a years who are ordering more takeout food than they did two years ago.

a. Create a scattergram of the data.
b. Which of the points in your scattergram is highest? What does that mean in this situation?
c. Which of the points in your scattergram is lowest? What does that mean in this situation?
d. Do the heights of the points in your scattergram increase or decrease from left to right? What does that mean in this situation?

Table 20 Percentages of Americans Who Are Ordering More Takeout Food than They Did Two Years Ago

Age Group (years)	Age Used to Represent Age Group (years)	Percent
18–24	21.0	34
25–34	29.5	31
35–44	39.5	27
45–54	49.5	17
55–64	59.5	15
over 64	70.0	7

Source: *National Restaurant Association Survey*

49. Several inventions are listed in Table 21, along with the years they were invented and how long it took for one-quarter of the U.S. population to use them ("mass use").

Table 21 Number of Years until Inventions Reached Mass Use

Invention	Year Invented	Years until Mass Use
Electricity	1873	46
Telephone	1876	35
Gasoline-Powered Automobile	1886	55
Radio	1897	31
Television	1923	29
Microwave Oven	1953	36
VCR	1965	13
Personal Computer	1975	16
Mobile Phone	1985	11
CD Player	1985	8
World Wide Web	1991	7
DVD Player	1997	5

Source: *Newsweek*

Let M be the number of years elapsed until an invention reached mass use if it was invented at t years since 1870.

a. Create a scattergram of the data.
b. Compare the time it took to reach mass use for recent inventions versus earlier inventions. In your opinion, why did this happen?
c. Does the datum for the microwave oven fit the pattern you described in part (b)? Explain.
d. For a while after the microwave oven was invented, many people feared that it would cause radiation poisoning, blindness, or impotence. Discuss the impact of these fears in terms of your response to part (c).
e. Explain why the datum for the gasoline-powered automobile does not fit the pattern you described in part (b). Why do you think this happened?

50. The percentages of adults of various age groups who approve of single men raising children on their own are shown in Table 22. Let p be the percentage of adults at age a years who approve of single men raising children on their own.

a. Create a scattergram of the data.
b. Which age group shown in Table 22 has the most faith in single men raising children on their own?
c. Which age group shown in Table 22 has the least faith in single men raising children on their own?

Table 22 Percentages of Adults Who Approve of Single Men Raising Children on Their Own

Age Group (years)	Age Used to Represent Age Group (years)	Percentage
18–34	26.0	81
35–44	39.5	73
45–54	49.5	73
55–64	59.5	66
over 64	70	47

Source: *Taylor Nelson Sofres*

51. The average starting salaries for employees with a bachelor's degree are illustrated in the bar graph in Fig. 29 for various fields of study (Source: *National Association of Colleges and Employers*).

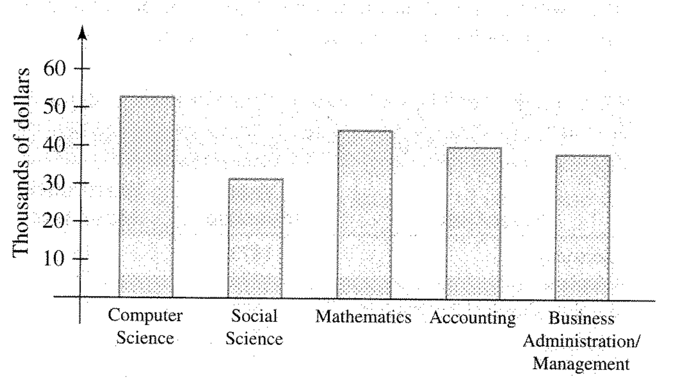

Figure 29 Average starting salaries

a. For which field shown is the average starting salary the highest? What is that salary?
b. For which field shown is the average starting salary the lowest? What is that salary?
c. Estimate the average beginning salary for employees with a mathematics degree.

52. In baseball, a grand slam is a home run with the bases loaded. The top five numbers of career grand slams for major league baseball players are illustrated in Fig. 30 (Source: *MLB.com*).
a. Estimate Robin Ventura's number of career grand slams.
b. Who has the record number of career grand slams? What is that number of grand slams?

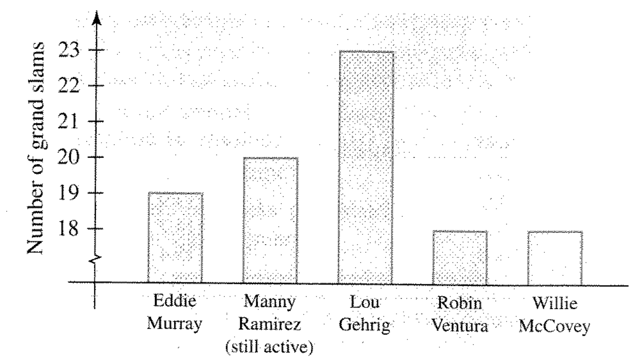

Figure 30 Most career grand slams

c. Of the five baseball players listed in Fig. 30, Manny Ramirez is the only active major league baseball player. Estimate how many grand slams he needs to tie the record.

53. List five ordered pairs whose x-coordinate is 3. Then create a scattergram of the ordered pairs. What do you notice about the arrangement of the points in your scattergram? Explain why this makes sense.

54. List five ordered pairs whose y-coordinate is 2. Then create a scattergram of the ordered pairs. What do you notice about the arrangement of the points in your scattergram? Explain why this makes sense.

55. The points where the sides of a triangle, rectangle, or any other polygon meet are called *vertices*. A square has vertices at (2, 1) and (2, 5). How many possible positions are there for the other two vertices? Find the coordinates of the verticies for two of these possible positions.

56. The points where the sides of a triangle, rectangle, or any other polygon meet are called *vertices*. A rectangle has vertices at (2, 4) and (7, 4). How many possible positions are there for the other two vertices? Find the coordinates of the verticies for four of these possible positions.

57. Describe the signs of the x-coordinate and the y-coordinate for a point that lies in the given quadrant.
a. Quadrant I
b. Quadrant II
c. Quadrant III
d. Quadrant IV

58. Compare a number line with a coordinate system. When is it useful to describe data by a number line? When is it useful to describe data by a coordinate system?

59. Compare the meaning of *dependent variable* with the meaning of *independent variable*.

1.3 EXACT LINEAR RELATIONSHIPS

If we did the things we are capable of, we would astound ourselves.

—Thomas Edison

Objectives

- Know the meaning of *linearly related, model, linear model, input,* and *output*.
- Use a linear model to make estimates and predictions.
- Use a scattergram to help decide whether to model a situation with a linear model.
- Know the meaning of *intercept*.
- Find intercepts of a line and of a linear model.

In this section, we will use a scattergram to help us sketch a line that can be used to describe an authentic situation, such as the descent of a hot-air balloon. We will then use the line to make estimates and predictions about the situation.

Linear Models

Example 1 Using a Line to Describe an Authentic Situation

A person lowers her hot-air balloon by gradually releasing air from the balloon. Let a be the balloon's altitude (in feet) above the ground after she has released air in the balloon for t minutes. Values of t and a are listed in Table 23.

Table 23 Altitudes of a Balloon

Time (minutes) t	Altitude (feet) a
0	2200
2	1800
4	1400
6	1000
8	600

1. Create a scattergram of the data.
2. Draw the line that contains the points of the scattergram.

Solution

1. We draw a scattergram in Fig. 31.

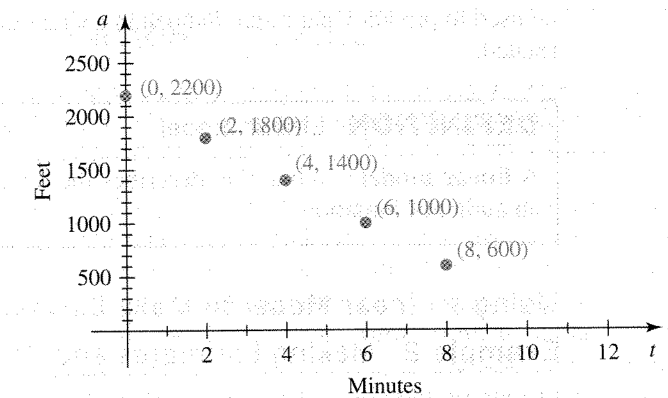

Figure 31 Scattergram of balloon data

2. In mathematics, a "line" means a *straight* line. In Fig. 32, we draw the line that contains the data points shown in Fig. 31.

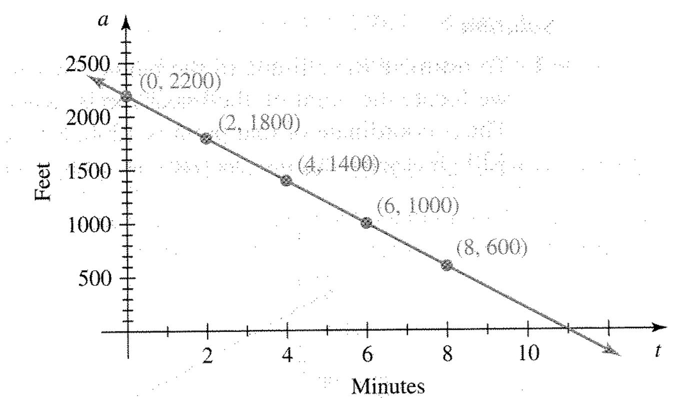

Figure 32 Line that contains the points of the scattergram

The scattergram in Fig. 31 accurately describes the altitude of the balloon at 0, 2, 4, 6, and 8 minutes. But it does not describe the altitude at other times.

If we imagine the line in Fig. 32 to be made up of points, then we can use the line to describe the altitudes at 0, 2, 4, 6, and 8 minutes accurately. We can also use the line to estimate the altitude for other times between 0 and 8 minutes. These results will be good estimates, provided that the altitude of the balloon declined steadily.

In addition, we can use the line to predict altitudes for times a little after 8 minutes. However, these predictions will be accurate only if the altitude of the balloon continued to decline steadily.

If the altitudes of the balloon over a span of time are described accurately by a line, we say that time and altitude (and the variables t and a) are *linearly related* for that span of time.

> **DEFINITION Linearly related**
>
> If two quantities of an authentic situation are described accurately by a line, then the quantities (and the variables representing those quantities) are **linearly related.**