

**Table 22** Percentages of Adults Who Approve of Single Men Raising Children on Their Own

Age Group (years)	Age Used to Represent Age Group (years)	Percentage
18–34	26.0	81
35–44	39.5	73
45–54	49.5	73
55–64	59.5	66
over 64	70	47

Source: Taylor Nelson Sofres

51. The average starting salaries for employees with a bachelor's degree are illustrated in the bar graph in Fig. 29 for various fields of study (Source: *National Association of Colleges and Employers*).

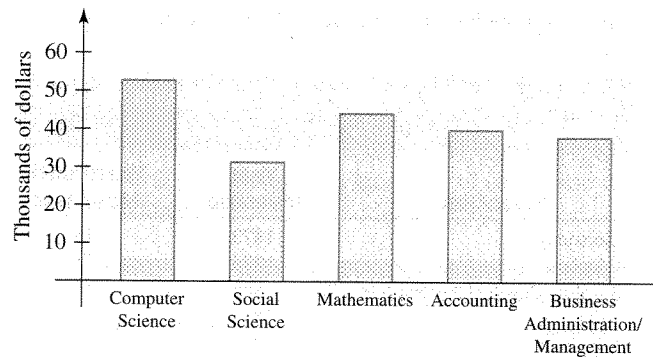


Figure 29 Average starting salaries

- For which field shown is the average starting salary the highest? What is that salary?
  - For which field shown is the average starting salary the lowest? What is that salary?
  - Estimate the average beginning salary for employees with a mathematics degree.
52. In baseball, a grand slam is a home run with the bases loaded. The top five numbers of career grand slams for major league baseball players are illustrated in Fig. 30 (Source: *MLB.com*).
- Estimate Robin Ventura's number of career grand slams.
  - Who has the record number of career grand slams? What is that number of grand slams?

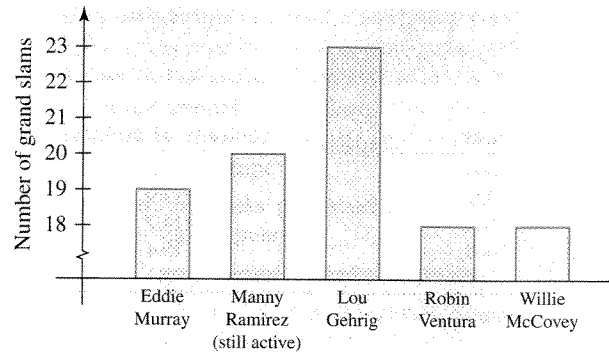


Figure 30 Most career grand slams

- Of the five baseball players listed in Fig. 30, Manny Ramirez is the only active major league baseball player. Estimate how many grand slams he needs to tie the record.
- List five ordered pairs whose  $x$ -coordinate is 3. Then create a scattergram of the ordered pairs. What do you notice about the arrangement of the points in your scattergram? Explain why this makes sense.
  - List five ordered pairs whose  $y$ -coordinate is 2. Then create a scattergram of the ordered pairs. What do you notice about the arrangement of the points in your scattergram? Explain why this makes sense.
  - The points where the sides of a triangle, rectangle, or any other polygon meet are called *vertices*. A square has vertices at (2, 1) and (2, 5). How many possible positions are there for the other two vertices? Find the coordinates of the vertices for two of these possible positions.
  - The points where the sides of a triangle, rectangle, or any other polygon meet are called *vertices*. A rectangle has vertices at (2, 4) and (7, 4). How many possible positions are there for the other two vertices? Find the coordinates of the vertices for four of these possible positions.
  - Describe the signs of the  $x$ -coordinate and the  $y$ -coordinate for a point that lies in the given quadrant.
    - Quadrant I
    - Quadrant II
    - Quadrant III
    - Quadrant IV
  - Compare a number line with a coordinate system. When is it useful to describe data by a number line? When is it useful to describe data by a coordinate system?
  - Compare the meaning of *dependent variable* with the meaning of *independent variable*.

### 1.3 EXACT LINEAR RELATIONSHIPS

*If we did the things we are capable of, we would astound ourselves.*

—Thomas Edison

#### Objectives

- Know the meaning of *linearly related*, *model*, *linear model*, *input*, and *output*.
- Use a linear model to make estimates and predictions.
- Use a scattergram to help decide whether to model a situation with a linear model.
- Know the meaning of *intercept*.
- Find intercepts of a line and of a linear model.

In this section, we will use a scattergram to help us sketch a line that can be used to describe an authentic situation, such as the descent of a hot-air balloon. We will then use the line to make estimates and predictions about the situation.

### Linear Models

#### Example 1 Using a Line to Describe an Authentic Situation

A person lowers her hot-air balloon by gradually releasing air from the balloon. Let  $a$  be the balloon's altitude (in feet) above the ground after she has released air in the balloon for  $t$  minutes. Values of  $t$  and  $a$  are listed in Table 23.

- Create a scattergram of the data.
- Draw the line that contains the points of the scattergram.

#### Solution

- We draw a scattergram in Fig. 31.

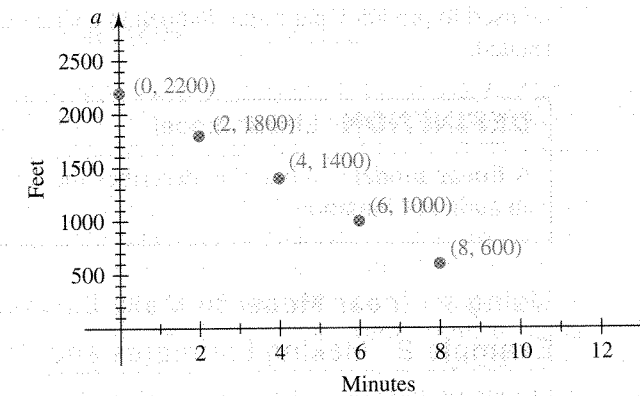


Figure 31 Scattergram of balloon data

- In mathematics, a "line" means a *straight* line. In Fig. 32, we draw the line that contains the data points shown in Fig. 31.

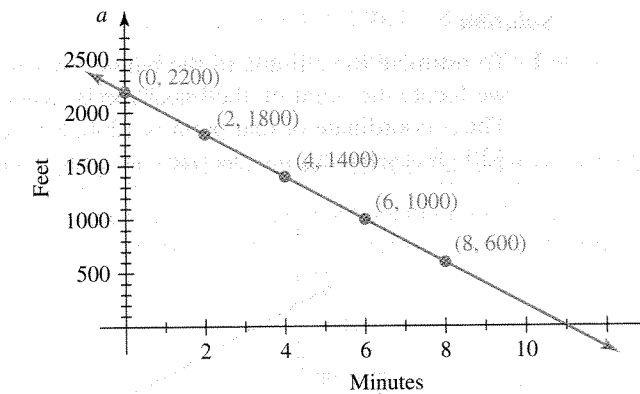


Figure 32 Line that contains the points of the scattergram

The scattergram in Fig. 31 accurately describes the altitude of the balloon at 0, 2, 4, 6, and 8 minutes. But it does not describe the altitude at other times.

If we imagine the line in Fig. 32 to be made up of points, then we can use the line to describe the altitudes at 0, 2, 4, 6, and 8 minutes accurately. We can also use the line to estimate the altitude for other times between 0 and 8 minutes. These results will be good estimates, provided that the altitude of the balloon declined steadily.

In addition, we can use the line to predict altitudes for times a little after 8 minutes. However, these predictions will be accurate only if the altitude of the balloon continued to decline steadily.

If the altitudes of the balloon over a span of time are described accurately by a line, we say that time and altitude (and the variables  $t$  and  $a$ ) are *linearly related* for that span of time.

#### DEFINITION Linearly related

If two quantities of an authentic situation are described accurately by a line, then the quantities (and the variables representing those quantities) are **linearly related**.

The process of choosing a line to represent the relationship between balloon altitudes and time is an example of *modeling*.

**DEFINITION Model**

A **model** is a mathematical description of an authentic situation.

We call the line in Fig. 32 a *linear model*. In Chapters 7–11, we will discuss other types of models. The term “model” is being used in much the same way as it is used in “airplane model.” Just as an airplane designer can use the behavior of an airplane model in a wind tunnel to predict the behavior of an actual airplane, a linear model can be used to predict what might happen in a situation in which two variables are linearly related.

**DEFINITION Linear model**

A **linear model** is a line that describes the relationship between two quantities in an authentic situation.

**Using a Linear Model to Make Estimates and Predictions**

**Example 2 Making Estimates and Predictions**

1. Use the linear model shown in Fig. 32 to estimate the balloon’s altitude when air has been released for 5 minutes.
2. Use the linear model to predict when the balloon’s altitude is 400 feet.

**Solution**

1. To estimate the altitude of the balloon when air has been released for 5 minutes, we locate the point on the linear model where the  $t$ -coordinate is 5 (see Fig. 33). The  $a$ -coordinate of that point is 1200. So, *according to the model*, the altitude is 1200 feet when air has been released for 5 minutes.

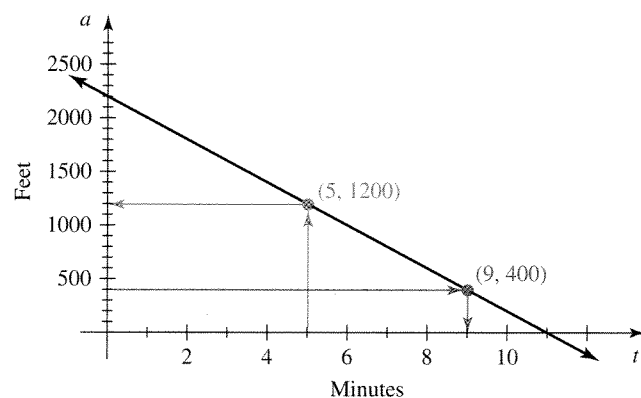


Figure 33 Using the linear model to make an estimate and a prediction

Table 24 Altitudes of a Balloon

Time (minutes)	Altitude (feet)
$t$	$a$
0	2200
2	1800
4	1400
6	1000
8	600

2. To find when the balloon’s altitude is 400 feet, we locate the point on the linear model where the  $a$ -coordinate is 400 (see Fig. 33). The  $t$ -coordinate of that point is 9. So, *according to the linear model*, the altitude is 400 feet when air has been released for 9 minutes.

Again, we verify our work by checking that our result is consistent with the values in Table 24. Since the altitude is 600 feet when air has been released for 8 minutes, it follows that air would have been released for more than 8 minutes for the altitude to be less than 600 feet, which checks with our result of 9 minutes. ■

**Input and Output**

In Example 2, we found that when the value of the independent variable  $t$  is 5, the corresponding value of the dependent variable  $a$  is 1200. We say that the *input* 5 leads to the *output* 1200. The blue arrows in Fig. 33 show the action of the input  $t = 5$  leading to the output  $a = 1200$ .

**DEFINITION Input, output**

An **input** is a permitted value of the *independent* variable that leads to at least one **output**, which is a permitted value of the *dependent* variable.

For a value to be permitted, it must make physical sense and be defined. For instance, in Example 2 the value  $-50$  is not a permitted value of the variable  $a$ , because it does not make sense for the balloon’s altitude to be  $-50$  feet. Later in the course we will discuss values that are not permitted for mathematical reasons.

Sometimes we will go “backward,” from an output back to an input. For instance, in Example 2 we found that the output  $a = 400$  originates from the input  $t = 9$ . The red arrows in Fig. 33 show the action of going backward from the output  $a = 400$  to the input  $t = 9$ .

**When to Use a Line to Model Data**

Next, we will discuss how to determine whether an authentic situation can be described well by a linear model.

**Example 3 Deciding whether to Use a Line to Model Data**

Consider the scattergrams of data for situations 1, 2, and 3 shown in Figs. 34, 35, and 36, respectively. For each situation, determine whether a linear model would describe the situation well.

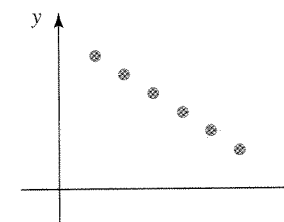


Figure 34 Scattergram for situation 1

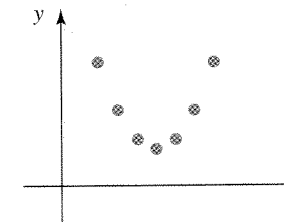


Figure 35 Scattergram for situation 2

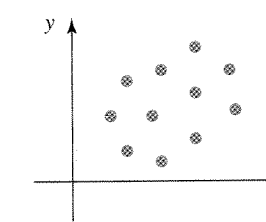


Figure 36 Scattergram for situation 3

**Solution**

It appears that the data points for situation 1 lie on a line; a linear model would describe situation 1 well. The data points for situation 2 do not lie close to one line; a linear model would not describe situation 2. (In Chapters 7–9, we will discuss a type of nonlinear model that would describe situation 2 well.) The data points for situation 3 do not lie near a line; a linear model would not describe this situation. ■

**We create a scattergram of data to determine whether the data points lie on a line. If the points lie on a line, then we draw the line and use it to make estimates and predictions.**

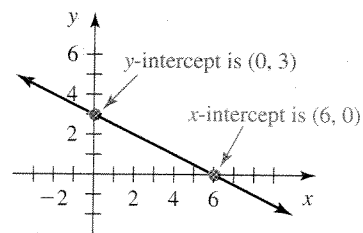


Figure 37 Intercepts of a line

### Intercepts of a Line

Consider the line sketched in Fig. 37. The line intersects the  $x$ -axis at the point  $(6, 0)$ . The point  $(6, 0)$  is called the  $x$ -intercept. Also, the line intersects the  $y$ -axis at the point  $(0, 3)$ . The point  $(0, 3)$  is called the  $y$ -intercept.

#### DEFINITION Intercepts of a line

An **intercept** of a line is any point where the line and an axis (or axes) of a coordinate system intersect. There are two types of intercepts of a line sketched on a coordinate system with an  $x$ -axis and a  $y$ -axis:

- An  **$x$ -intercept** of a line is a point where the line and the  $x$ -axis intersect (see Fig. 38). The  $y$ -coordinate of an  $x$ -intercept is 0.
- A  **$y$ -intercept** of a line is a point where the line and the  $y$ -axis intersect (see Fig. 38). The  $x$ -coordinate of a  $y$ -intercept is 0.

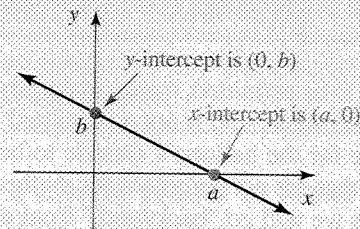


Figure 38 Intercepts of a line

### Example 4 Finding Intercepts and Coordinates

Refer to Fig. 39 for the following problems.

1. Find the  $x$ -intercept of the line.
2. Find the  $y$ -intercept of the line.
3. Find  $y$  when  $x = -6$ .
4. Find  $x$  when  $y = -3$ .

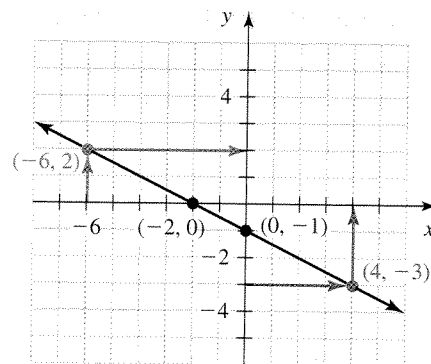


Figure 39 Problems 1–4 of Example 4

#### Solution

1. The line and the  $x$ -axis intersect at  $(-2, 0)$ . So, the  $x$ -intercept is  $(-2, 0)$ .
2. The line and the  $y$ -axis intersect at  $(0, -1)$ . So, the  $y$ -intercept is  $(0, -1)$ .
3. The blue arrows in Fig. 39 show that the input  $x = -6$  leads to the output  $y = 2$ . So,  $y = 2$  when  $x = -6$ .
4. The red arrows in Fig. 39 show that the output  $y = -3$  originates from the input  $x = 4$ . So,  $x = 4$  when  $y = -3$ .

### Intercepts of a Linear Model

Suppose that a linear model describes the relationship between two variables  $t$  and  $p$ , where  $p$  depends on  $t$ . Then the  $t$ -intercept is a point where the line and the  $t$ -axis intersect, and the  $p$ -intercept is a point where the line and the  $p$ -axis intersect (see Fig. 40).

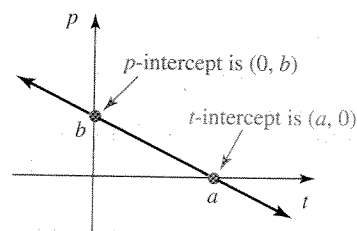


Figure 40 Intercepts of a linear model

### Example 5 Finding Intercepts of a Linear Model

1. The linear model from Examples 1 and 2 is shown in Fig. 41. Find the  $t$ -intercept of the model. What does it mean in this situation?
2. Find the  $a$ -intercept of the model. What does it mean in this situation?

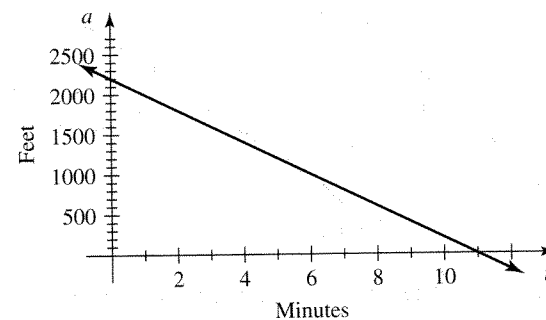


Figure 41 Altitudes of balloon model

#### Solution

1. The line intersects the  $t$ -axis at the point  $(11, 0)$ . See Fig. 42. So, the  $t$ -intercept is  $(11, 0)$ . This means that when  $t = 11$ ,  $a = 0$ . The model predicts that the balloon reached the ground 11 minutes after air began to be released from the balloon.

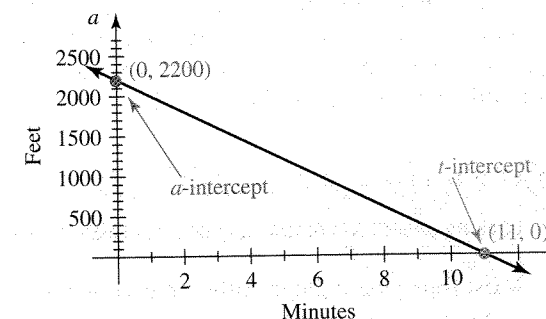


Figure 42 Intercepts of the linear model

2. The line intersects the  $a$ -axis at the point  $(0, 2200)$ . See Fig. 42. So, the  $a$ -intercept is  $(0, 2200)$ . This means that when  $t = 0$ ,  $a = 2200$ . The model estimates that the balloon's altitude was 2200 feet when air was first released from the balloon. In fact, this estimate is the actual altitude (see Table 24 on p. 26).

### Example 6 Using a Linear Model to Make Estimates

An underground rock band manufactures 500 CDs of its original music and tries to sell the CDs at the band's concerts. Let  $P$  be the profit (in dollars) from selling a total of  $n$  CDs. Some values of  $n$  and  $P$  are listed in Table 25.

Table 25 Profits from Selling CDs

Sales (number of CDs)	Profit (dollars)
$n$	$P$
50	-1150
100	-800
150	-450
300	600
350	950

1. Sketch a scattergram of the data. Then draw a reasonable model.
2. Estimate the band's profit from selling all 500 CDs.
3. If the band loses \$975, estimate how many CDs will have been sold.
4. Estimate the  $P$ -intercept of the model. What does it mean in this situation?
5. Estimate the  $n$ -intercept of the model. What does it mean in this situation?

#### Solution

1. The scattergram is shown in Fig. 43 (the black points). Since the points lie on a line, we use the line as a model.
2. The blue arrows show that the input  $n = 500$  leads to the output  $P = 2000$ . So, if the band sells 500 CDs, the profit will be \$2000.
3. A negative value of  $P$  represents a loss. The red arrows show that the output  $P = -975$  originates from the input  $n = 75$ . So, if the band sells 75 CDs, it will lose \$975.



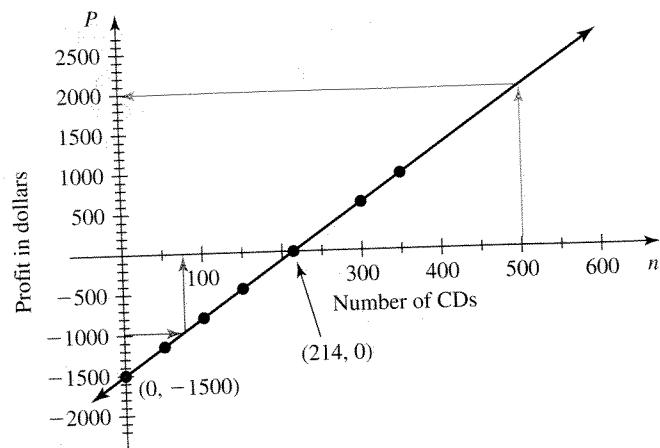
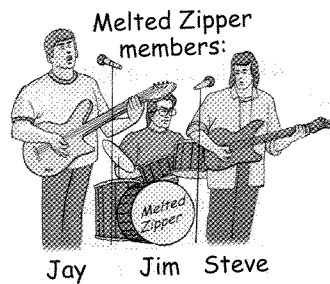


Figure 43 Scattergram and linear model of the CD situation



- The linear model and the  $P$ -axis intersect at the point  $(0, -1500)$ . This means that the band will lose \$1500 if no CDs are sold.
- The linear model and the  $n$ -axis intersect at about the point  $(214, 0)$ . This means that the band will have a profit of about 0 dollars (and hence break even) if 214 CDs are sold.

It is impossible to do a perfect job of sketching models and estimating coordinates of points. Your results for homework exercises will likely be different from the answers provided near the end of this textbook. However, if you do a careful job, your results should be close to those in the book.

## group exploration

Looking ahead: Approximately linearly related variables

Most four-year colleges offer an early-decision admissions process, in which students can apply early. By applying early, students can hear back from colleges earlier, which can lessen stress levels for those students who receive acceptance letters. Students who apply via a college's early-acceptance system must commit to enrolling in the college if they are accepted. In a seven-year study, researchers found that a student applicant has a better chance of being accepted to a college through early decision than by regular decision. The number of students who apply to college under early-decision plans has increased greatly since 1997 (see Table 26).

Let  $n$  be the number of students (in thousands) who applied to college under early-decision plans at  $t$  years since 1995.

- Create a scattergram of the data.
- Draw a line that comes close to all of the data points in your scattergram.
- Use your line to estimate the number of students who applied under early-decision programs in 2004.
- Use your line to predict when 100 thousand students will apply under early-decision programs.
- What is the  $n$ -intercept of the linear model? What does it mean in this situation?

Table 26 Students Who Applied via Early-Decision Plans

Year	Number of Students (thousands)
1997	42
1998	50
1999	54
2000	63
2001	67
2002	70
2003	79

Source: The College Board

### TIPS FOR SUCCESS: Get in Touch with Classmates

It is wise to exchange phone numbers and e-mail addresses with some classmates. If any of you has to miss class, then you have someone to contact to find out what you missed and what homework was assigned.

## HOMWORK 1.3 FOR EXTRA HELP



For Exercises 1–6, refer to Fig. 44.

- Find  $y$  when  $x = -2$ .
- Find  $y$  when  $x = 4$ .
- Find  $x$  when  $y = -2$ .
- Find  $x$  when  $y = 4$ .
- What is the  $x$ -intercept of the line?
- What is the  $y$ -intercept of the line?

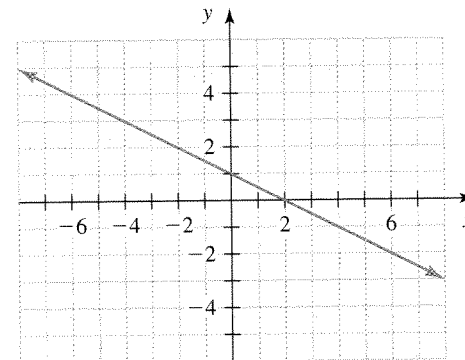


Figure 44 Exercises 1–6

For Exercises 7–12, refer to Fig. 45.

- Find  $y$  when  $x = -3$ .
- Find  $y$  when  $x = 6$ .
- Find  $x$  when  $y = -3$ .
- Find  $x$  when  $y = 0$ .
- What is the  $y$ -intercept of the line?
- What is the  $x$ -intercept of the line?

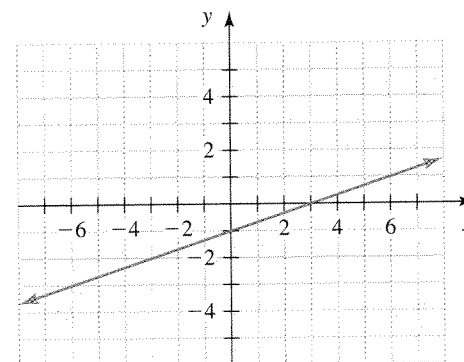


Figure 45 Exercises 7–12

13. Some ordered pairs are listed in Table 27.

- Create a scattergram of the points shown in Table 27.
- Draw a line that contains the points in your scattergram.
- Find  $y$  when  $x = 3$ .
- Find  $x$  when  $y = 6$ .

Table 27 Some Ordered Pairs

$x$	$y$
2	20
4	16
6	12
8	8
10	4

- What is the  $y$ -intercept of your line?
  - What is the  $x$ -intercept of your line?
14. Some ordered pairs are listed in Table 28.

Table 28 Some Ordered Pairs

$x$	$y$
-6	4
-2	8
2	12
6	16
10	20

- Create a scattergram of the points shown in Table 28.
  - Draw a line that contains the points in your scattergram.
  - Find  $y$  when  $x = 4$ .
  - Find  $x$  when  $y = 17$ .
  - What is the  $y$ -intercept of your line?
  - What is the  $x$ -intercept of your line?
15. Water is steadily pumped out of a flooded basement. Let  $v$  be the volume of water (in thousands of gallons) that remains in the basement  $t$  hours after water began to be pumped. A linear model is shown in Fig. 46.

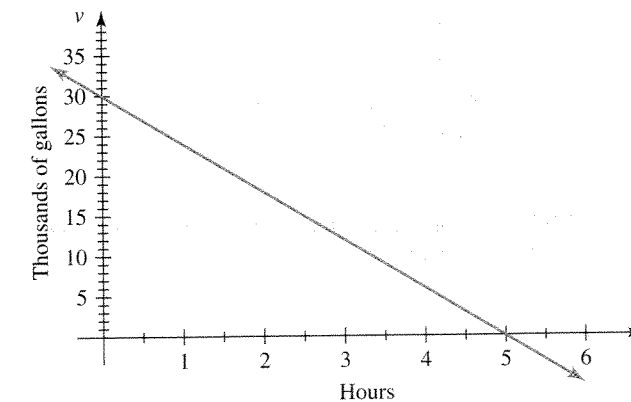


Figure 46 Linear model—Exercise 15

- How much water is in the basement after 2 hours of pumping?
- After how many hours of pumping will 5 thousand gallons remain in the basement?

- c. How much water was in the basement before any water was pumped out?  
 d. After how many hours of pumping will all the water be pumped out of the basement?
16. Let  $B$  be the balance (in dollars) of a student's checking account at  $t$  months since the student opened the account. A linear model is shown in Fig. 47.

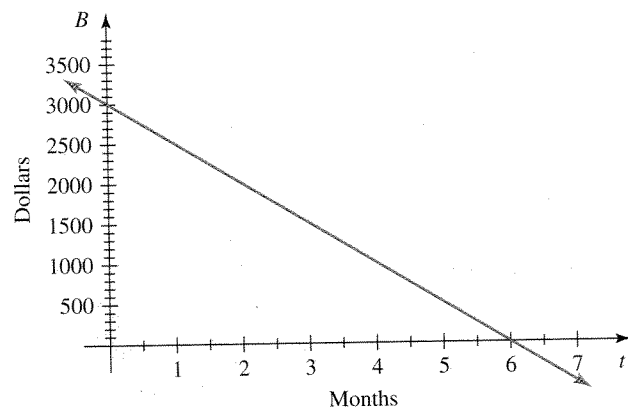


Figure 47 Linear model—Exercise 16

- a. What was the balance 3 months after the student opened the account?  
 b. When was the balance \$500?  
 c. What is the  $B$ -intercept of the model? What does it mean in this situation?  
 d. What is the  $t$ -intercept of the model? What does it mean in this situation?
17. A scattergram for a situation is graphed in Fig. 48. Is there a line that is a reasonable model of this situation? If yes, sketch the line. If no, explain why not.

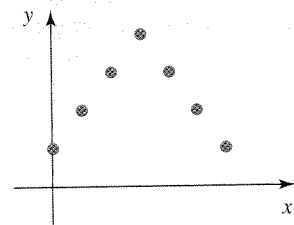


Figure 48 Scattergram—Exercise 17

18. A scattergram for a situation is graphed in Fig. 49. Is there a line that is a reasonable model of this situation? If yes, sketch the line. If no, explain why not.

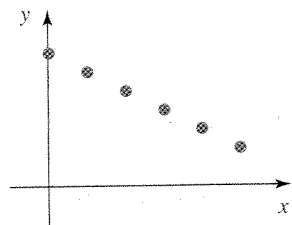


Figure 49 Scattergram—Exercise 18

19. Some ordered pairs are listed in Table 29.

Table 29 Some Ordered Pairs

$x$	$y$
0	10
5	5
8	2
10	1
12	2
13	5
14	10

- a. Create a scattergram of the data shown in Table 29.  
 b. Is there a linear relationship between  $x$  and  $y$ ? Explain.
20. Some ordered pairs are listed in Table 30.

Table 30 Some Ordered Pairs

$x$	$y$
2	1
3	6
4	9
5	10
6	9
7	6
8	1

- a. Create a scattergram of the data shown in Table 30.  
 b. Is there a linear relationship between  $x$  and  $y$ ? Explain.
21. Let  $d$  be the distance traveled (in miles) after a student has driven for  $t$  hours (not counting pit stops). Some pairs of values of  $t$  and  $d$  are shown in Table 31.

Table 31 Times and Distances for a Car

$t$ (hours)	$d$ (miles)
0	0
1	60
2	120
3	180
4	240

- a. Create a scattergram of the data. Then draw a linear model.  
 b. Estimate how far the student has traveled in 2.5 hours.  
 c. Estimate how long it took the student to travel 210 miles.
22. A student works part time at the college bookstore. Let  $p$  be the student's pay (in dollars) for working  $t$  hours. Some pairs of values of  $t$  and  $p$  are shown in Table 32.

Table 32 Pay for Working  $t$  Hours

$t$ (hours)	$p$ (dollars)
0	0
5	40
10	80
15	120
20	160

- a. Create a scattergram of the data. Then draw a linear model.  
 b. Estimate the student's pay for working 7 hours.  
 c. Estimate the number of hours the student must work to earn \$96.

23. Let  $E$  be a college's enrollment (in thousands of students) at  $t$  years since the college began. Some pairs of values of  $t$  and  $E$  are shown in Table 33.

Table 33 Ages and Enrollments of a College

$t$ (years)	$E$ (thousands of students)
0	5
1	7
2	9
3	11
4	13

- a. Create a scattergram of the data. Then draw a linear model.  
 b. Predict the enrollment when it has been 6 years since the college opened.  
 c. Predict when the enrollment will reach 19 thousand students.
24. Let  $s$  be a person's salary (in thousands of dollars) after he has worked  $t$  years at a company. Some pairs of values of  $t$  and  $s$  are shown in Table 34.

Table 34 Years Worked and Salary

$t$ (years)	$s$ (thousands of dollars)
0	20
2	24
4	28
6	32
8	36

- a. Create a scattergram of the data. Then draw a linear model.  
 b. Estimate the person's salary after he has worked 5 years at the company.  
 c. Estimate when the person's salary will be \$34 thousand.  
 d. What is the  $s$ -intercept of the model? What does it mean in this situation?
25. Let  $v$  be the value (in dollars) of a company's stock at  $t$  years since 2000. Some pairs of values of  $t$  and  $v$  are shown in Table 35.

Table 35 Values of a Stock

$t$ (years)	$v$ (dollars)
1	28
2	24
4	16
6	8
7	4

- a. Create a scattergram of the data. Then draw a linear model.  
 b. Estimate when the value of the stock was \$12.  
 c. What is the  $t$ -intercept of the model? What does it mean in this situation?  
 d. What is the  $v$ -intercept of the model? What does it mean in this situation?
26. Let  $p$  be the profit (in millions of dollars) of a company for the year that is  $t$  years since 2000. Some pairs of values of  $t$  and  $p$  are shown in Table 36.

Table 36 Profits of a Company

$t$ (years)	$p$ (millions of dollars)
1	20
2	18
3	16
5	12
6	10

- a. Create a scattergram of the data. Then draw a linear model.  
 b. Predict when the profit will be \$2 million.  
 c. What is the  $p$ -intercept of the model? What does it mean in this situation?  
 d. What is the  $t$ -intercept of the model? What does it mean in this situation?
27. Let  $g$  be the number of gallons of gasoline that remain in a car's gasoline tank after the car has been driven  $d$  miles since the tank was filled. Some pairs of values of  $d$  and  $g$  are shown in Table 37.

Table 37 Miles Traveled and Gallons of Gasoline

$d$ (miles)	$g$ (gallons)
40	11
80	9
120	7
160	5
200	3
240	1

- a. Create a scattergram of the data. Then draw a linear model.  
 b. Estimate how much gasoline is in the tank after the driver has gone 140 miles since last filling up.  
 c. Estimate the number of miles driven since the tank was last filled if 2 gallons of gasoline remain in the tank.  
 d. Find the  $d$ -intercept of the model. What does it mean in this situation?  
 e. Find the  $g$ -intercept of the model. What does it mean in this situation?
28. Let  $v$  be the value (in thousands of dollars) of a car when it is  $t$  years old. Some pairs of values of  $t$  and  $v$  are listed in Table 38.

Table 38 Ages and Values of a Car

$t$ (years)	$v$ (thousands of dollars)
1	18
3	14
5	10
7	6
9	2

- a. Create a scattergram of the data. Then draw a linear model.  
 b. Estimate the age of the car when it is worth \$4 thousand.  
 c. Estimate the value of the car when it is 6 years old.  
 d. What is the  $v$ -intercept of the model? What does it mean in this situation?  
 e. What is the  $t$ -intercept of the model? What does it mean in this situation?



29. Let  $r$  be the revenue (in millions of dollars) of a company for the year that is  $t$  years since 2000. Some pairs of values of  $t$  and  $r$  are shown in Table 39.

Table 39 Revenues of a Company

$t$ (years)	$r$ (millions of dollars)
0	8
1	11
3	17
5	23
6	26

- a. Create a scattergram of the data. Then draw a linear model.  
 b. Predict the revenue in 2010.  
 c. Estimate when the revenue was \$14 million.  
 d. What is the  $r$ -intercept of the model? What does it mean in this situation?
30. Let  $v$  be the value (in dollars) of a company's stock at  $t$  years since 2000. Some pairs of values of  $t$  and  $v$  are shown in Table 40.

Table 40 Values of a Stock

$t$ (years)	$v$ (dollars)
0	15
2	19
4	23
5	25
6	27

- a. Create a scattergram of the data. Then draw a linear model.  
 b. Estimate the value of the stock in 2003.  
 c. Predict when the value of the stock will be \$35.  
 d. What is the  $v$ -intercept of the model? What does it mean in this situation?

31. Let  $a$  be the altitude (in thousands of feet) of an airplane at  $t$  minutes since the airplane began its descent. Some pairs of values of  $t$  and  $a$  are shown in Table 41.

Table 41 Altitudes of an Airplane

$t$ (minutes)	$a$ (thousands of feet)
0	36
5	30
10	24
15	18
20	12

- a. Create a scattergram of the data. Then draw a linear model.  
 b. Use your model to estimate the airplane's altitude 12 minutes after it began its descent.  
 c. Use your model to estimate when the airplane will reach the ground.  
 d. Assume that your line does a good job of modeling the airplane's descent up until the last 2000 feet, at which point the airplane then descends at a slower rate than before. Is your estimate in part (c) an underestimate or an overestimate? Explain.

32. Let  $a$  be the altitude (in feet) of a hot-air balloon after the air in the balloon is released for  $t$  minutes. Some pairs of values of  $t$  and  $a$  are shown in Table 42.

Table 42 Altitudes of a Balloon

$t$ (minutes)	$a$ (feet)
0	1800
1	1600
3	1200
4	1000
6	600

- a. Create a scattergram of the data. Then draw a linear model.  
 b. Estimate the balloon's altitude after air has been released for 5 minutes.  
 c. Estimate when the balloon will reach the ground.  
 d. Assume that your line does a good job of modeling the balloon's descent up until the last 400 feet, at which point the balloon then descends at a faster rate than before. Is your estimate in part (c) an underestimate or an overestimate? Explain.

33. Let  $p$  be the percentage of major U.S. firms that perform drug tests on employees and/or job applicants at  $t$  years since 1980. Some pairs of values of  $t$  and  $p$  are shown in Table 43.

Table 43 Percentages of Firms That Perform Drug Tests

$t$ (years)	$p$ (percent)
7	22
10	51
12	72
15	78
18	74
21	67

Source: American Management Association

- a. Create a scattergram of the data.  
 b. Are the variables  $t$  and  $p$  linearly related? Explain.

34. Let  $p$  be the percentage of flights that are delayed at  $t$  years since 1995. Some pairs of values of  $t$  and  $p$  are shown in Table 44.

Table 44 Percentages of Flights That Are Delayed

$t$ (years)	$p$ (percent)
3	21
4	22
5	24
6	20
7	17
8	16
9	20
10	21

Source: Bureau of Transportation Statistics

- a. Create a scattergram of the data.  
 b. Are the variables  $t$  and  $p$  linearly related? Explain.

35. A student says that the  $y$ -intercept of the ordered pair  $(2, 5)$  is 5. Is the student correct? Explain.  
 36. A student says that the  $x$ -intercept of the ordered pair  $(-3, 4)$  is  $-3$ . Is the student correct? Explain.  
 37. A student says that the  $x$ -intercept of a line is  $(0, 2)$ . Is the student correct? Explain.  
 38. A student says that the  $y$ -intercept of a line is  $(5, 0)$ . Is the student correct? Explain.  
 39. A student says that the  $x$ -intercept of a line is 5. Is the student correct? Explain.  
 40. Are there any lines for which the  $x$ -intercept is the same point as the  $y$ -intercept? If yes, sketch such a line, and what is that point? If no, explain why not.  
 41. Sketch three distinct lines that all have the same  $x$ -intercept.

42. Sketch three distinct lines that all have the same  $y$ -intercept.  
 43. a. Sketch a nonvertical line in a coordinate system. Find any outputs for the given input. State how many outputs there are for that single input.  
     i. the input 2   ii. the input 4   iii. the input  $-3$   
 b. For your line, a single input leads to how many outputs? Explain.  
 c. For any nonvertical line, a single input leads to how many outputs? Explain.  
 44. In your own words, describe the meaning of *linear model*. (See page 9 for guidelines on writing a good response.)  
 45. Describe how to find a linear model of a situation and how to use the model to make estimates and predictions. (See page 9 for guidelines on writing a good response.)

## 1.4 APPROXIMATE LINEAR RELATIONSHIPS

### Objectives

- ▶ Know the meaning of *approximately linearly related*.
- ▶ Use a linear model to make estimates and predictions.
- ▶ Find errors in estimations.
- ▶ Know the meaning of *model breakdown*.

In this section, we will use a line to model a situation in which data points lie close to the line, but not necessarily on the line.

### Modeling when Variables Are Approximately Linearly Related

#### Example 1 Using a Line to Model Data

The average ticket prices for the top-50-grossing concert tours are shown in Table 45 for various years. Let  $p$  be the average ticket price (in dollars) at  $t$  years since 1995. Sketch a scattergram of the data, and draw a line that comes close to the points of the scattergram.

#### Solution

First, we list values of  $t$  and  $p$  in Table 46. For example,  $t = 3$  represents 1998, because 1998 is 3 years after 1995; and  $t = 4$  represents 1999, because 1999 is 4 years after 1995.

Next, we sketch a scattergram in Fig. 50. It makes sense to think of  $p$  as the dependent variable, so we let the vertical axis be the  $p$ -axis. Since  $t$  is the independent variable, the horizontal axis is the  $t$ -axis.

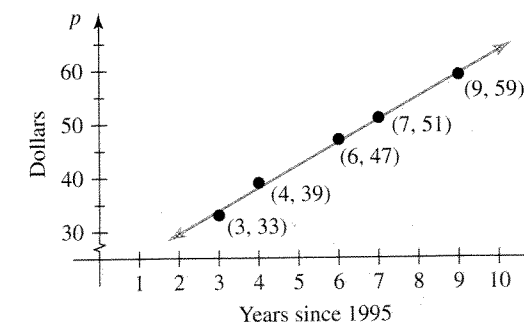


Figure 50 Average ticket price scattergram, and model

Then we sketch a line that comes close to points of the scattergram (see Fig. 50).

Table 45 Average Ticket Prices for Top-50-Grossing Concert Tours

Year	Average Ticket Price (dollars)
1998	33
1999	39
2001	47
2002	51
2004	59

Source: Pollstar

Table 46 Using Values of  $t$  to Stand for the Years

Number of Years since 1995	Average Ticket Price (dollars)
$t$	$p$
3	33
4	39
6	47
7	51
9	59