Example 3

How much 20% alcohol solution and 50% alcohol solution must be mixed in order to make 12 gallons of 30% alcohol solution?

Solution: Like the previous problem this involves percentages of different quantities and we can relate the quantity of alcohol with its percent concentration. e.g., 8 gallons of 20% alcohol contains 8 • 0.2 = 1.6 gallons of alcohol.

Include columns for d, r, and t

	Quantity of mix	Concentration	Quantity of alc.
20% solution			
50% solution			
Total			

→

Begin by filling in what you *know*:

	Quantity of mix	Concentration	Quantity of alc.
20% solution		0.2	
50% solution		0.5	
Total	12	0.3	3.6

Then fill in what you *want* :

	Quantity of mix	Concentration	Quantity of alc.
20% solution	x	0.2	
50% solution	y	0.5	
Total	12	0.3	3.6

Let x = quantity of 20% solution.
Let y = quantity of 50% solution.

→

Complete the Alcohol column:

	Quantity of mix	Concentration	Quantity of alc.
20% solution	x	0.2	$0.2x$
50% solution	y	0.5	$0.5y$
Total	12	0.3	3.6

↓ ↓

$x + y = 12$ $\quad$ $0.2x + 0.5y = 3.6$

Then we have an equation for the total quantity of solution and total quantity of alcohol:

This gives us the system of equations:

$$\begin{aligned} x + y &= 12 \\ 0.2x + 0.5y &= 3.6 \end{aligned}$$

Solving the first for y: $y = 12 - x$ and substituting into the second equation gives $\to 0.2x + 0.5(12 - x) = 3.6$
Then $0.2x + 6 - 0.5x = 3.6 \to -0.3x = -2.4 \to x = 8$ gallons of 20% solution. Then $y = 12 - 8 = 4$ gall. of 50%.

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Example 1

A boat travels 30 miles up a river (against the current) in 5 hours. The boat returns to its starting place on the river (with the current) in 3 hours. What is the speed of the boat in still water? What is the speed of the current?

Solution: Since this involves distance, rates, and time we plan to use $d = r \cdot t$. There are two different relations involved (d, r, t up river, and d, r, t down river) so let's begin with a table:

Include columns for d, r, and t:

	r	t	$d = rt$
Upstream			
Downstream			

→

Begin by filling in what you *know*:

	r	t	$d = rt$
Upstream		5	
Downstream		3	

Then fill in what you *want* :

	r	t	$d = rt$
Upstream	$x - y$	5	
Downstream	$x + y$	3	

Let x = speed in still water and
let y = speed of the current

→

Complete the d column using $d = r \cdot t$:

	r	t	$d = rt$
Upstream	$x - y$	5	$5(x - y)$
Downstream	$x + y$	3	$3(x + y)$

We know the distance is the same for both (30 miles) so we complete the last column with:

$$5(x - y) = 30$$
$$3(x + y) = 30$$

This gives us the system:

$$5x - 5y = 30 \xrightarrow{\times 3} 15x - 15y = 90$$
$$3x + 3y = 30 \xrightarrow{\times 5} 15x + 15y = 150$$
$$30x = 240$$

So $x = 8$ mph
and $3(8) + 3y = 30$
so $3y = 6 \rightarrow y = 2$ mph.

It follows that the boat goes 8mph in still water and the current is 2 mph.

Example 2

You want to invest $5000 but while you want to make money, you are concerned about taking a risk. A friend suggests you split the investment - part at a higher risk 9% rate and the rest at a conservative 5%. If your goal is to make $350, how much should you invest at each rate?

Solution: Since this problem involves Principal, Rate, and interest we will use $I = P \cdot r$. Since it involves two different rates (and two different amounts) we will keep track with a table:

Include columns for I, P, and r:

	P	r	$I = P \cdot r$
Invest @ 9%			
Invest @ 5%			
Total		–	

→

Begin by filling in what you *know*:

	P	r	$I = P \cdot r$
Invest @ 9%		0.09	
Invest @ 5%		0.05	
Total		–	

Then fill in what you *want* :

	P	r	$I = P \cdot r$
Invest @ 9%	x	0.09	
Invest @ 5%	y	0.05	
Total	5000	–	

Let x = principal invested at 9% and
y = principal invested at 5%.

→

Complete the I column using $I = P \cdot r$

	P	r	$I = P \cdot r$
Invest @ 9%	x	0.09	$.09x$
Invest @ 5%	y	0.05	$.05y$
Total	$5000	–	$350

We have two totals and two sets of expressions to equate to them: $x + y = 5000$ $\quad .09x + .05y = 350$

Setting up the system gives: $\begin{aligned} x + y &= 5000 \\ .09x + .05y &= 350 \end{aligned}$ and we can solve by either elimination or substitution.

The top equation is perfect for substitution so we solve for y: $y = 5000 - x$ and substitute into the other equation:

$$.09x + .05(5000 - x) = 350 \rightarrow .09x - .05x + 250 = 350 \rightarrow .04x = 100 \rightarrow x = \$2500$$
$$y = 5000 - x \rightarrow y = \$2500$$

So you invest $2500 at 5% and $2500 at 9% in order to make $350 in interest.