### Solutions

**You may use a calculator to verify solutions, but not to provide them.**

1. Solve:
   
   (a) \[ x^2 = 5x + 14 \]
   \[ x^2 - 5x - 14 = 0 \]
   \[ (x - 7)(x + 2) = 0 \]
   \( x - 7 = 0 \) and \( x + 2 = 0 \)
   \( x = 7 \) and \( x = -2 \)

   (b) \[ 16x^2 - 25 = 0 \]
   \[ (4x - 5)(4x + 5) = 0 \]
   \[ 4x - 5 = 0 \] and \( 4x + 5 = 0 \)
   \( x = 5/4 \) and \( x = -5/4 \)

   (c) \[ x^2 - x = \frac{1}{5} \]
   \[ \frac{x^2}{5} - \frac{x}{2} = \frac{1}{5} \]
   \[ 10 \left( \frac{x^2}{5} - \frac{x}{2} \right) = 10 \left( \frac{1}{5} \right) \]
   \[ 2x^2 - 5x = -2 \]
   \[ 2x^2 - 5x + 2 = 0 \]
   \[ (2x - 1)(x - 2) = 0 \]
   \( x = 1/2 \) and \( x = 2 \)

2. Write a quadratic equation for which \( x = -4 \) and \( x = \frac{3}{2} \) are solutions.

   **Solution:** The factored form might look like \( (x + 4)(x - \frac{3}{2}) = 0 \). However, to write this without fractions we can take a hint from problems like 1(b) above and write it as \( (x + 4)(2x - 3) = 0 \).

3. Write an equation of a parabola for which \( x = -4 \) and \( x = \frac{3}{2} \) are the \( x \)-intercepts.

   **Solution:** This is similar to (2) but the equation of a parabola is a function so the \( x \)-intercepts are just the special case where \( y = 0 \). Therefore our answer is \( y = (x + 4)(2x - 3) \) or, if we distribute it, \( y = 2x^2 + 5x - 12 \).

4. Write an equation of a *different* parabola for which \( x = -4 \) and \( x = \frac{3}{2} \) are the \( x \)-intercepts.

   **Solution:** Anything of the form \( y = k(x + 4)(2x - 3) \) will work here, since the \( x \)-intercepts remain the same.
   The distinction is that as you change \( k \), the steepness of the parabola changes – or it flips, if you use \( k < 0 \).

5. Find the point symmetric with the \( y \)-intercept of the parabola \( y = x^2 - 7x + 5 \).

   **Solution:** The \( y \)-intercept is \( (0, 5) \) so the symmetric point will be at the other solution to \( x^2 - 7x + 5 = 5 \).
   Solving gives us:
   \[ x^2 - 7x + 5 = 5 \]
   \[ x^2 - 7x = 0 \]
   \[ x(x - 7) = 0 \]
   \( x = 0 \) and \( x = 7 \)
6. The graph of \( y = -x^2 + x + 6 \) is shown to right. Find the values of the intercepts \( k, m, \) and \( n \) and the coordinates of the vertex (the high point), without a calculator.

**Solution:** \( k \) is the \( y \)-intercept so we know \( x = 0 \) and it follows that \( k = -(0)^2 + 0 + 6 = 6 \)

The \( x \)-intercepts, \( m \) and \( n \) occur where \( y = 0 \) so

\[
\begin{align*}
-x^2 + x + 6 &= 0 \\
-1(-x^2 + x + 6) &= -1(0) \\
x^2 - x - 6 &= 0 \\
(x + 2)(x - 3) &= 0 \\
x &= -2 & \text{and} & \quad x &= 3
\end{align*}
\]

So \( m = -2 \) and \( n = 3 \).

The vertex occurs between any two symmetric points so if we average the \( x \)-intercepts we get the \( x \)-coordinate of the vertex: \( x = \frac{-2 + 3}{2} = \frac{1}{2} \).

The \( y \)-coordinate comes from plugging \( x \) into the original equation: \( y = -(\frac{1}{2})^2 + \frac{1}{2} + 6 = 6\frac{1}{4} \). Therefore the vertex is at \( (\frac{1}{2}, \frac{25}{4}) \).

7. The graph of a parabola of the form \( y = ax^2 + bx + c \) is shown to right. Find the equation of this parabola using the given intercepts.

**Solution:** From the graph we know the parabola has \( x \)-intercepts at \( x = 2 \) and \( x = 3 \) so it has factors \( (x - 2)(x - 3) \). From \#4 above, we have seen the general form of this parabola will be \( y = k(x - 2)(x - 3) \). Since the \( y \)-intercept is at \( (0, 12) \), we know that when \( x = 0 \) in our equation we should have \( y = 12 \) so

\[
\begin{align*}
12 &= k(0 - 2)(0 - 3) \\
12 &= 6k \\
2 &= k
\end{align*}
\]

Then we have \( y = 2(x - 2)(x - 3) \) or \( y = 2x^2 - 10x + 12 \).