Solutions

Show all relevant work!

YOU MAY USE A CALCULATOR TO VERIFY SOLUTIONS, BUT NOT TO PROVIDE THEM.

1. Solve:

(a)

$$\begin{aligned} x^2 &= 5x + 14 \\ x^2 - 5x - 14 &= 0 \\ (x-7)(x+2) &= 0 \\ x - 7 = 0 &\quad \text{and} \quad x + 2 = 0 \\ x = 7 &\quad \text{and} \quad x = -2 \end{aligned}$$

(b)

$$\begin{aligned} 16x^2 - 25 &= 0 \\ (4x-5)(4x+5) &= 0 \\ 4x - 5 = 0 &\quad \text{and} \quad 4x + 5 = 0 \\ x = \frac{5}{4} &\quad \text{and} \quad x = -\frac{5}{4} \end{aligned}$$

(c)

$$\begin{aligned} \frac{x^2}{5} - \frac{x}{2} &= -\frac{1}{5} \\ 10\left(\frac{x^2}{5} - \frac{x}{2}\right) &= 10\left(-\frac{1}{5}\right) \\ 2x^2 - 5x &= -2 \\ 2x^2 - 5x + 2 &= 0 \\ (2x-1)(x-2) &= 0 \\ x = \frac{1}{2} &\quad \text{and} \quad x = 2 \end{aligned}$$

2. Write a quadratic equation for which $x = -4$ and $x = \frac{3}{2}$ are solutions.

Solution: The factored form might look like $(x+4)(x-\frac{3}{2}) = 0$. However, to write this without fractions we can take a hint from problems like 1(b) above and write it as $(x+4)(2x-3) = 0$.

3. Write *an* equation of a parabola for which $x = -4$ and $x = \frac{3}{2}$ are the x–intercepts.

Solution: This is similar to (2) but the equation of a parabola is a function so the x–intercepts are just the special case where $y = 0$. Therefore our answer is $y = (x+4)(2x-3)$ or, if we distribute it, $y = 2x^2 + 5x - 12$.

4. Write an equation of a *different* parabola for which $x = -4$ and $x = \frac{3}{2}$ are the x–intercepts.

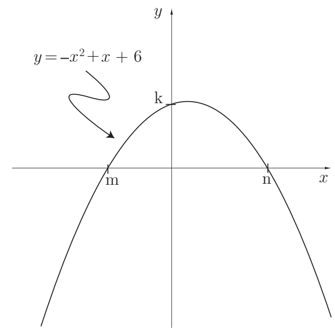
Solution: Anything of the form $y = k(x+4)(2x-3)$ will work here, since the x–intercepts remain the same. The distinction is that sa you change k, the steepness of the parabola changes – or it flips, if you use $k < 0$.

5. Find the point symmetric with the y-intercept of the parabola $y = x^2 - 7x + 5$.

Solution: The y–intercept is $(0, 5)$ so the symmetric point will be at the other solution to $x^2 - 7x + 5 = 5$. Solving gives us:

$$\begin{aligned} x^2 - 7x + 5 &= 5 \\ x^2 - 7x &= 0 \\ x(x-7) &= 0 \\ x = 0 &\quad \text{and} \quad x = 7 \end{aligned}$$

6. The graph of $y = -x^2 + x + 6$ is shown to right.
Find the values of the intercepts k, m, and n and the coordinates of the vertex (the high point), without a calculator.



Solution: k is the y–intercept so we know $x = 0$ and it follows that $k = -(0)^2 + 0 + 6 = 6$

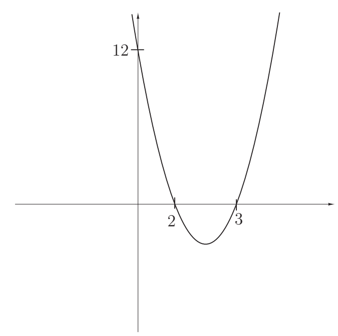
The x–intercepts, m and n occur where $y = 0$ so

$$\begin{aligned} -x^2 + x + 6 &= 0 \\ -1\left(-x^2 + x + 6\right) &= -1(0) \\ x^2 - x - 6 &= 0 \\ (x+2)(x-3) &= 0 \\ x = -2 \quad &\text{and} \quad x = 3 \end{aligned}$$

So $m = -2$ and $n = 3$.

The vertex occurs between any two symetric points so if we average the x–intercepts we get the x–coordinate of the vertex: $x = \frac{-2+3}{2} = \frac{1}{2}$.
The y–coordinate comes from plugging x into the original equation: $y = -(\frac{1}{2})^2 + \frac{1}{2} + 6 = 6\frac{1}{4}$.
Therefore the vertex is at $(\frac{1}{2}, 6\frac{1}{4})$.

7. The graph of a parabola of the form $y = ax^2 + bx + c$ is shown to right.
Find the equation of this parabola using the given intercepts.



Solution: From the graph we know the parabola has x–intercepts at $x = 2$ and $x = 3$ so it has factors $(x-2)(x-3)$. From #4 above, we have seen the general form of this parabola will be $y = k(x-2)(x-3)$. Since the y–intercept is at (0, 12), we know that when $x = 0$ in our equation we should have $y = 12$ so

$$\begin{aligned} 12 &= k(0-2)(0-3) \\ 12 &= 6k \\ 2 &= k \end{aligned}$$

Then we have $y = 2(x-2)(x-3)$ or $y = 2x^2 - 10x + 12$.