

Identity Proofs

Name: _____

Definitions:

$$\sin \theta = \frac{y}{r}$$

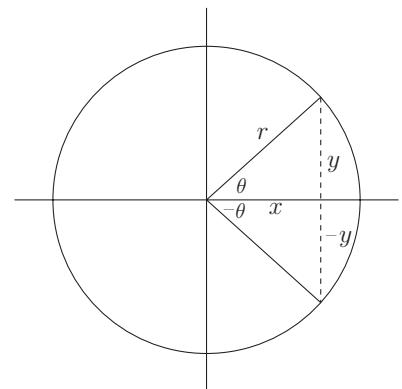
$$\cos \theta = \frac{x}{r}$$

$$\tan \theta = \frac{y}{x}$$

$$\csc \theta = \frac{r}{y}$$

$$\sec \theta = \frac{r}{x}$$

$$\cot \theta = \frac{x}{y}$$

**Reciprocal Identities:**

$$\csc \theta = \frac{1}{\sin \theta}$$

$$\sec \theta = \frac{1}{\cos \theta}$$

$$\cot \theta = \frac{1}{\tan \theta}$$

Odd/Even Identities:

$$\sin(-\theta) = -\sin \theta$$

$$\cos(-\theta) = \cos \theta$$

$$\tan(-\theta) = -\tan \theta$$

Fundamental Identities:

$$1. \tan \theta = \frac{\sin \theta}{\cos \theta} \longrightarrow \cot \theta = \frac{\cos \theta}{\sin \theta}$$

$$2. \sin^2 \theta + \cos^2 \theta = 1 \longrightarrow \tan^2 \theta + 1 = \sec^2 \theta, \quad 1 + \cot^2 \theta = \csc^2 \theta$$

Sum and Difference Identities:

$$1. \cos(\theta - \phi) = \cos \theta \cos \phi + \sin \theta \sin \phi$$

$$5. \tan(\theta + \phi) = \frac{\tan \theta + \tan \phi}{1 - \tan \theta \tan \phi}$$

$$2. \cos(\theta + \phi) = \cos \theta \cos \phi - \sin \theta \sin \phi$$

$$6. \tan(\theta - \phi) = \frac{\tan \theta - \tan \phi}{1 + \tan \theta \tan \phi}$$

$$3. \sin(\theta - \phi) = \sin \theta \cos \phi - \sin \phi \cos \theta$$

$$4. \sin(\theta + \phi) = \sin \theta \cos \phi + \sin \phi \cos \theta$$

Double Angle Identities:

$$1. \sin(2\theta) = 2 \sin \theta \cos \theta$$

$$3. \cos(2\theta) = 1 - 2 \sin^2 \theta$$

$$2. \cos(2\theta) = \cos^2 \theta - \sin^2 \theta$$

$$4. \cos(2\theta) = 2 \cos^2 \theta - 1$$

Half Angle Identities:

$$1. \sin\left(\frac{\alpha}{2}\right) = \pm \sqrt{\frac{1 - \cos \alpha}{2}}$$

$$2. \cos\left(\frac{\alpha}{2}\right) = \pm \sqrt{\frac{1 + \cos \alpha}{2}}$$

$$3. \tan\left(\frac{\alpha}{2}\right) = \pm \sqrt{\frac{1 - \cos \alpha}{1 + \cos \alpha}}$$