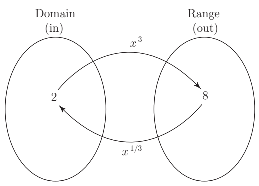
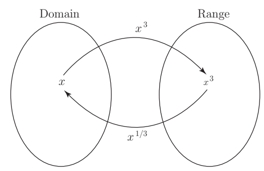
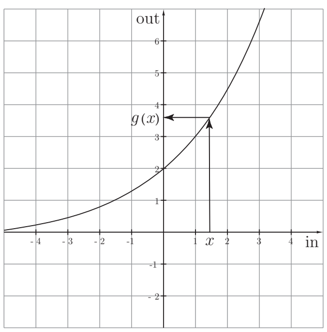
Inverse Functions

In order to reverse the effect of adding a number, we subtract it. To reverse multiplication, we divide. Similarly, inverse functions reverse the action of the original function. For example, $y = x^3$ and $y = \sqrt[3]{x}$ are inverses.

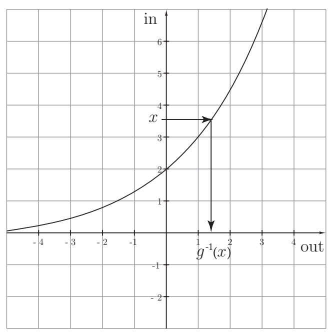




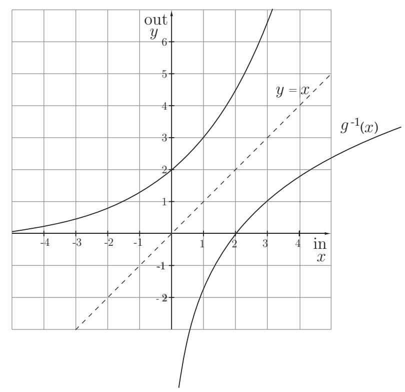
As we've seen with inverse functions in general, there are different views of the graphical representation:



$g(x) = 2(1.5)^x$



$g^{-1}(x)$
Axes reversed



$g(x), g^{-1}(x)$
Axes correct

The graph of $g(x) = 2(1.5)^x$ reminds us of the general shape of an exponential function. The center graph shows the idea of inverting $g(x) = 2(1.5)^x$ by reversing the input and output axes. Switching the in and out values themselves will produce the graph of the inverse function in the usual orientation. This gives some sense of what the general shape of the inverse of an exponential function looks like.

Similarly, we can make the inverse of a function table simply by reversing the input and output values.

x (in)	-3	-1	0	2	5
$f(x)$ (out)	7	2	-1	-3	4

out	7	2	-1	-3	4
in	-3	-1	0	2	5

x (new in)	7	2	-1	-3	4
$f^{-1}(x)$ (new out)	-3	-1	0	2	5

Consider this when you want to graph the inverse of a function.

Exercises:

$\boxed{1.}$ The following table shows values for the function f.

x	-6	-3	-1	0	1	4	7
$f(x)$	-6	-5	0	2	4	7	9

(a) Find $f(4)$: ________________

(b) Solve $f(x) = 4$: ________________

(c) Solve $f(x) = x$: ________________

(d) Find $f^{-1}(9)$: ________________

(e) Solve $f^{-1}(x) = 0$: ________________

$\boxed{2.}$ Use the exponential function, $g(x)$, graphed below to answer the given questions.

(a) Estimate $g(0)$.

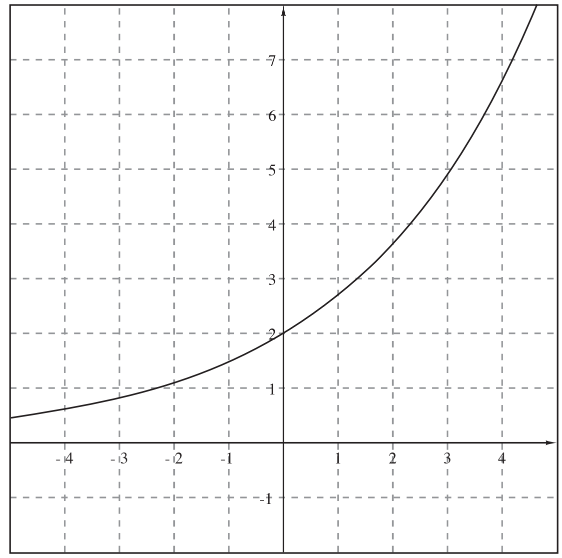
(b) Estimate $g(2)$.

(c) Estimate $g^{-1}(1)$.

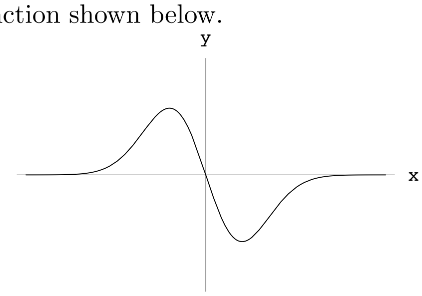
(d) Estimate $g^{-1}(5)$.

(e) Estimate $g^{-1}(5.5)$.

(f) Sketch the graph of $g^{-1}(x)$ on the same axes. (Make a table first).



$\boxed{3.}$ On the same set of axes, sketch the inverse of the function shown below.



$\boxed{4.}$ If $g(x) = 2^x + \sqrt{x} - 1$, find $g^{-1}(17)$.

$\boxed{5.}$ Suppose $P = f(t)$ gives the population of a city in millions of people as a function of time in years since 1950.
(a) Give the units and interpret the meaning of $f(30) = 9.2$.

(b) Give the units and interpret the meaning of $f^{-1}(15) = 49$.

$\boxed{6.}$ Find the inverse function for $f(x) = \dfrac{x+1}{x-1}$.