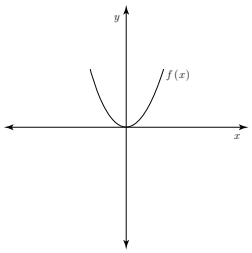
Some Investigations

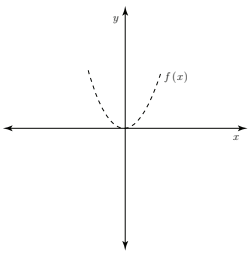
0.1 Shifts and Stretches

Recall the various manipulations of a function and their related effects. Complete the sketches and tables for each manipulation.

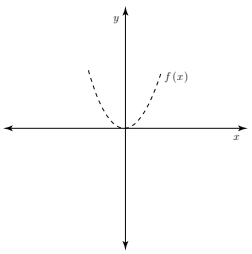
Shifts (vertical)
graphic:



$f(x)$



$f(x)+a$
$(a>0)$



$f(x)-a$
$(a>0)$

tabular:

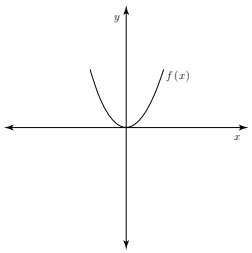
x	-2	-1	0	1	2
$f(x)$	4	1	0	1	4

x	-2	-1	0	1	2
$f(x)+a$					

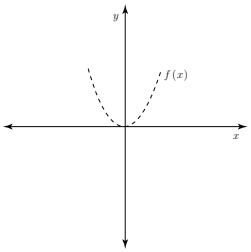
x	-2	-1	0	1	2
$f(x)-a$					

(e.g. $a=2$)

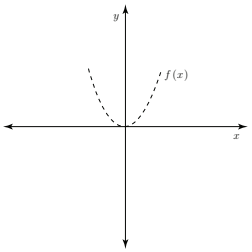
Shifts (horizontal)
graphic:



$f(x)$



$f(x+a)$
$(a>0)$



$f(x-a)$
$(a>0)$

tabular:

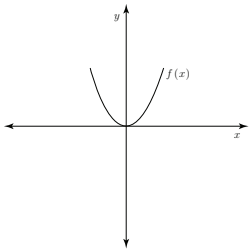
x	-2	-1	0	1	2
$f(x)$	4	1	0	1	4

x	-2	-1	0	1	2
$f(x+a)$					

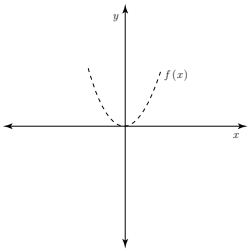
x	-2	-1	0	1	2
$f(x-a)$					

(e.g. $a=2$)

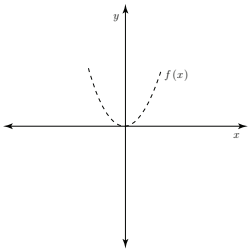
Reflections (vertical) (horizontal)
graphic:



$f(x)$



$-f(x)$



$f(-x)$

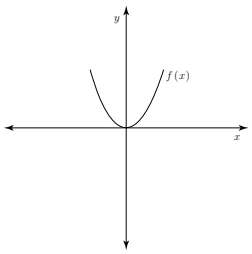
tabular:

x	-2	-1	0	1	2
$f(x)$	4	1	0	1	4

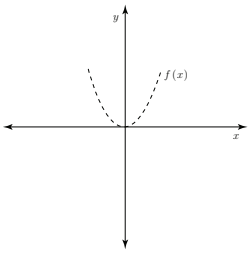
x	-2	-1	0	1	2
$-f(x)$					

x	-2	-1	0	1	2
$f(-x)$					

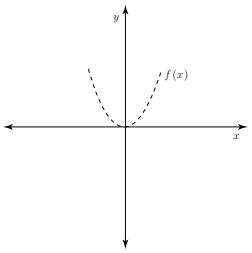
Stretches/Compressions (vertical)
graphic:



$f(x)$



$af(x)$
$(a>1)$



$af(x)$
$(0<a<1)$

tabular:

x	-2	-1	0	1	2
$f(x)$	4	1	0	1	4

x	-2	-1	0	1	2
$af(x)$					

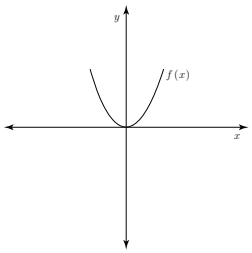
(e.g. $a=2$)

x	-2	-1	0	1	2
$af(x)$					

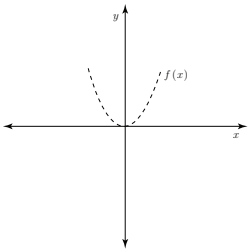
(e.g. $a=\frac{1}{2}$)

Stretches/Compressions (horizontal)

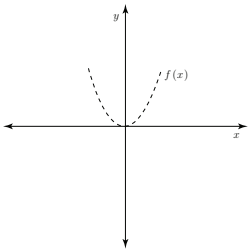
graphic:



$f(x)$



$f(ax)$
$(a > 1)$



$f(ax)$
$(0 < a < 1)$

tabular:

x	-2	-1	0	1	2
$f(x)$	4	1	0	1	4

x	-2	-1	0	1	2
$f(ax)$					

(e.g. $a = 2$)

x	-2	-1	0	1	2
$f(ax)$					

(e.g. $a = \frac{1}{2}$)

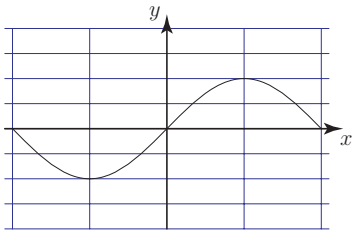
In each of the preceding cases, discuss the effects of the shift or stretch on the average rate of change of the function on the interval $[-2, 2]$.

0.2 Odd & Even Functions

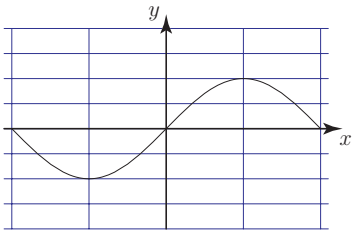
Sketch the following functions.

1. Sketch the indicated reflection over the graph of $g(x)$ in each exercise.

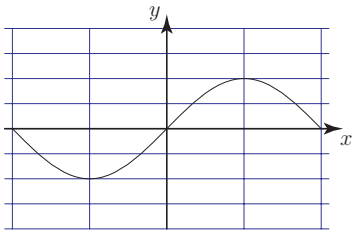
(a) $-f(x)$



(b) $f(-x)$

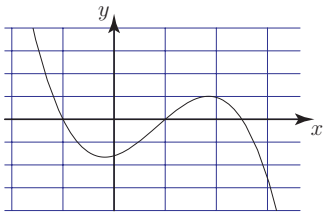


(c) $-f(-x)$

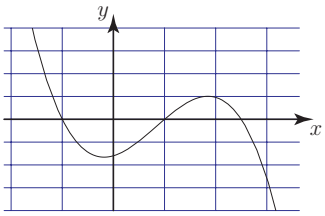


2. Sketch the indicated function over the graph of $f(x)$ in each exercise.

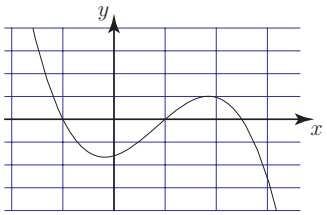
(a) $-f(x)$



(b) $f(-x)$

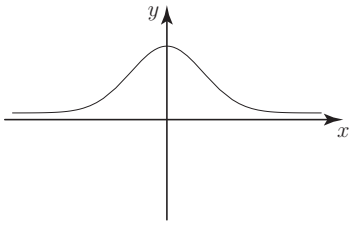


(c) $-f(-x)$

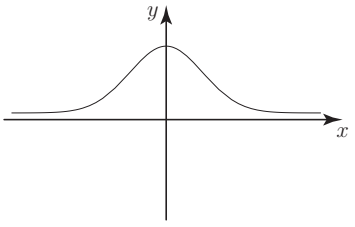


3. Sketch the indicated function over the graph of $f(x)$ in each exercise.

(a) $-f(x)$



(b) $f(-x)$



(c) $-f(-x)$