Projectile Motion

Mathematics is inadequate to describe the universe, since mathematics is an abstraction from natural phenomena.
Also, mathematics may predict things which don't exist, or are impossible in nature. – Ludovico delle Colombe Criticizing Galileo.

0.1 Equipment

- Card stock or binder
- Meter stick
- Mini Launcher and ball
- Plunger
- Carbon paper and white paper

0.2 Purpose

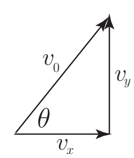
The objective for this experiment is to develop and apply a mathematical model for the position of a steel ball after it is launched into the air.

0.3 Theory

We've derived formulas to model horizontal (1) and vertical (2) motion, where g reflects acceleration due to gravity, v_x and v_y represent velocity in the horizontal and vertical directions respectively.

$$x(t) = v_x t \quad (1)$$
$$h(t) = -\frac{1}{2}gt^2 + v_y t + h_0 \quad (2)$$

We then generalized these into vector component formulas (3 and 4) based on an initial velocity, v_0 and launch angle, θ.



$$x(t) = (v_0 \cos\theta)t \quad (3)$$
$$h(t) = -\frac{1}{2}gt^2 + (v_0 \sin\theta)t + h_0 \quad (4)$$

By solving (4) for the time the projectile is in the air and substituting in (3) we derived the range formula relating the horizontal distance, the launch angle, and the initial velocity:

$$x(t) = (v_0 \cos\theta)\left(\frac{v_0 + \sqrt{v_0^2 + 2gh_0}}{g}\right) \quad (5)$$

0.4 Procedure

0.4.1 Determining Initial velocity

In order to approximate the initial velocity, we'll use two different methods and combine their results.

Method 1

1. Clamp the Mini Launcher near one end of the table.
2. Adjust the angle of the launcher to 90° so that the ball is launched straight up.
3. Place a meter stick vertically next to the launcher.
4. Put the ball into the Mini Launcher and cock it to the long range position. You will need one person to catch the ball on its way back down and another to sight the high spot on the meter stick. Take a trial shot to see how high the ball goes and to determine where to stand in order to read the meter stick without getting hit. *Remember* to catch the ball before it falls back onto the launcher.
5. Once you've determined how to measure the high point in the ball's path, make several shots and record the maximum heights in the table:
6. Average the heights and subtract the distance from the ground to the launch stripe marked on the side of the barrel. Determine the initial velocity.

Trial:	1	2	3	4	5	6
Height (m):						

Method 2

7. Take down the meter stick.
8. This time you will measure the time between when the ball is launched and when it returns to the height of the launcher. Your launcher should still be directed straight up.
9. Start the stopwatch as you pull the launch trigger (the person pulling the trigger should also be the person with the stopwatch).
10. Immediately after launch, place a binder or piece of cardstock over the muzzle (to protect the launcher) and stop the watch when the ball strikes the binder.
11. Repeat the launch several times and record the results below.

Trial:	1	2	3	4	5	6
Time (s):						

12. Use your results from the two methods to determine a reasonable approximation of v_0.

0.4.2 Predicting the Range of the Ball Shot at an Angle

1. Adjust the launcher to launch between 20° and 60° above horizontal.
2. Record this angle and use it along with the velocity you calculated above to determine where the ball will land. Remember it will land on the floor so you will need to include the height of the launcher above the ground in your calculation.
3. Place a binder or card stock on the floor where you expect the ball to hit and fire away. Note how accurate your shot was (you can tape a piece of carbon paper to the floor near your binder and use the mark to determine accuracy).
4. Repeat this exercise for several different angles and record your results.

Angle (°):					
Expected Distance (m):					
Actual Distance (m):					

5. **Bonus:** Ask me to mark a target. Calculate the angle necessary to hit the target and demonstrate the accuracy of your methods by successfully hitting the target.