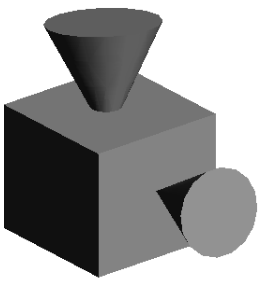
Math 241

Notes for Chapter 1.1

Announce: Solutions manual and other books on reserve.

Overview in brief: The primary goal of statistics and most applied math is to take empirical data (numbers collected by observation) , to recognize a pattern in the graph or table, and to fit an appropriate formula to the data so that we can make predictions about future and past behavior.

I. Functions.



If we did it right in algebra 1 & 2 you know a function is a relationship between two quantities. It may be expressed as pairs (or more) of numbers in a table, a graph, related by a formula ($y = 3x - 5$), or a situation (the lower the price, the more we sell). Ideally you should be able to move seamlessly between each of these representations – and that's one of the main goals for this course.

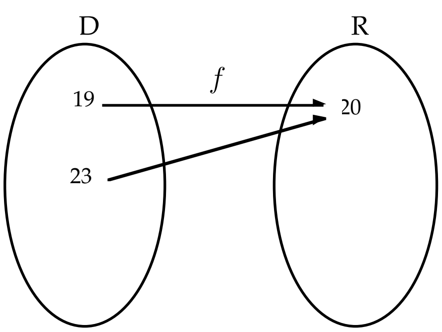
Most applied mathematics can be neatly modeled using functions.
Specifically, a function is a rule that takes certain numbers as inputs and assigns to each input a single output number. The set of input numbers is called the **domain** while the set of output numbers is called the **range**.

Ex. Consider the high temperature in Chicago each day for 10 days in December 00:

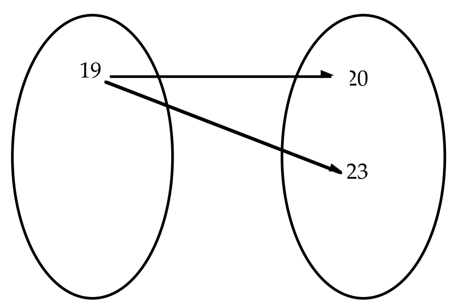
Date (Dec/00)	19	20	21	22	23	24	25	26	27	28	← domain
High temp	20	17	19	7	20	11	17	19	17	20	← range

(Sketch graph)

You can drop in different objects (values) and the machine f may crank out the same value, **but** a single value will never produce two different outcomes from the function. What does this tell you about the graph?



Yes



No

Defining a function with a table is OK, occasionally that's all that's available, but then the domain is limited to the **sample** values. If we want numbers in between, we have to *interpolate*.

II. Speaking of formulas . . . (an applied example)

The Snow Tree Cricket. For the entire population the chirp rate is the same at each particular temperature. By collecting data at different temperatures biologists created the regression formula (you'll do this shortly) $C = 4T - 160$ where C is chirps per minute and T is temperature.

We say that since C depends on T, the number of chirps per minute is a *function* of temperature.
In function notation, this reads $C = f(T)$.

See pg 16 #43, 44

Whenever we define a function formula we provide two pieces of information:

(1) Formula (2) Domain (input set)

For the Snow Tree Cricket example, $C = 4T - 160$, notice that if $T = 0$ we get $C = -160$ chirps per minute which has very little validity. As a result, we have to limit the values we can put in so that the numbers that come out are meaningful. For this model the minimum chirp rate should be ?<0>

Therefore we need the temperature to be $T \geq 40$ (right?)

As for the upper limit, this is usually found empirically but we can probably assume that 500° isn't too likely and in this case 130° will do.

We write the domain: $\{T\!: 40 \leq T \leq 130\}$ or [40, 130]

Using a table or formula to guess outcomes beyond domain or table values is called extrapolation – beware.

If no domain is specified, we assume the domain is **R**.

Find the domain for these functions:

a) $f(x) = \dfrac{1}{x+2}$ b) $f(x) = \dfrac{1}{x^2-9}$ c) $f(x) = \sqrt{x^2-9}$

So always look out for situations with $f = \frac{1}{?}$ and $f = \sqrt{?}$

III. The benefit of function notation is it's easy to express substitution:

If $f(x) = x^2$ then $f(3) =$______, so $f(a) =$______, and $f(a+b) =$______.

One of the appealing aspects of the TI-83 is how it evaluates functions:

If $f(x) = 0.3x^2 - 2x + 1.9$ and you want to find f(1.1)
we can input this in y1 and evaluate (see notes).
See table values.

IV. Piecewise functions . . .

Many situations seem to work well with one function for a while and then another function later on. Car sales might look like this – where each line segment has a different formula. Epidemics (AIDS for example) are often modeled this way. This is called a *piecewise* function.

Ex.

$$f(x)\begin{cases} 2x+1 & x \leq 3 \\ 5x-8 & x > 3 \end{cases}$$

Find $f(1),\ f(5)$