Math 241

Exam 3 Name:

Show all relevant work!

YOU MAY USE A CALCULATOR TO VERIFY SOLUTIONS, BUT NOT TO PROVIDE THEM.

1. Find the derivative of each function.

(a) $y = \frac{x-1}{x^2-x}$ $= \frac{\cancel{x-1}}{x(\cancel{x-1})}$ $= \frac{1}{x}$

6 $\boxed{y' = -\frac{1}{x^2}}$

$\frac{1(x^2-x) - (2x-1)(x-1)}{(x^2-x)^2}$

(b) $y = \frac{1}{x \ln x}$

$y = (x \ln x)^{-1}$

$y' = -(x \ln x)^{-2} \cdot (\ln x + 1)$

$= \boxed{-\frac{\ln x + 1}{(x \ln x)^2}}$

(c) $g(x) = \ln \sqrt{x^2+1}$

$= \frac{1}{2} \ln(x^2+1)$

$y' \Rightarrow \frac{1}{2} \cdot \frac{1}{x^2+1} \cdot 2x$

$= \boxed{\frac{x}{x^2+1}}$

2. Kate invests \$2000 at 5% compounded quarterly.

(a) Find the formula for the balance of the account as a function of time.

$B = 2000\left(1 + \frac{.05}{4}\right)^{4t}$

(b) By what percentage each year (APR) is Kate's money increasing?

$B = 2000(1 + .0125)^{4t}$

$= 2000(1.0509)^t \longrightarrow$ ~~APR~~ $\approx 5.1\%$

(c) How quickly is the amount of money in Kate's account increasing when $t = 6$ years?

$\frac{dB}{dt} = 2000 \cdot \ln(1.0509) \cdot (1.0509)^6$

$= 2000 \cdot (0.0497)(1.0509)^6$

$\approx \$133.90/\text{yr}.$

3. The deer population of an island is modeled by the logistic function $P(t) = \frac{70}{1+15e^{-0.34t}}$, where t is measured in years.

a) How quickly is the population increasing when $t = 9$ years?

$$P(t) = 70(1 + 15e^{-0.34t})^{-1}$$

$$P'(t) = -70(1+15e^{-0.34t})^{-2} \cdot -0.34 \cdot 15e^{-0.34t}$$

$$= \frac{357e^{-0.34t}}{(1+15e^{-0.34t})^2}$$

$$P'(9) = \frac{357e^{-0.34(9)}}{(1+15e^{-0.34 \cdot 9})^2} \approx 5.769$$

ABOUT 6 deer/yr.

b) Consider the graph of P. What value does $\frac{dP}{dt}$ approach as $t \to \infty$?

$$\frac{dP}{dt} \to 0.$$

4. The daily cost, C, of running an air conditioner in Arizona depends on the temperature, H, as shown in the first table. The temperature in turn increases with the time of day, t, as shown in the second table. Determine the rate at which cost changes with time when $t = 10$ and interpret the result.

H (in F°)	90	95	100	105	110
$C(H)$ (\$)	4	4.75	6	7.50	9.15

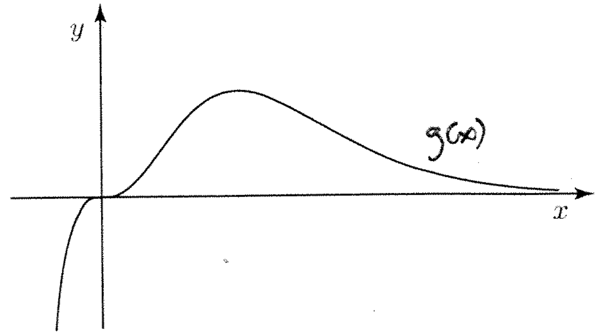
t (in hours past 00:00)	6	8	10	12	14
$H(t)$ (in F°)	90	97	100	112	119

$$\frac{dC}{dt} = \frac{dC}{dH} \cdot \frac{dH}{dt}$$

$$\approx \frac{7.5-4.75}{10} \cdot \frac{112-97}{4}$$

$$\approx \$1.031/\text{hr}$$

5. The graph of $g(x) = x^3e^{-x}$ is shown below. Your friend Duane tells you that even though the function appears to approach $y = 0$, he thinks that it may have a local maximum somewhere farther down the x–axis. What would you tell Duane about the local extremes of g? Provide support for your argument.



$g'(x) = 3x^2e^{-x} - x^3e^{-x}$

$= x^2e^{-x}(3-x)$

SINCE $g'(x) = 0$ AT $x = 0$ AND $x = 3$ (& NO POINTS WHERE UNDEF)

THE ONLY LOCAL EXTREMA OF g COULD OCCUR AT

$x = 0$ AND $x = 3$.

$g(0) = 0$ IS NEITHER MAX NOR MIN

$g(3) = \frac{27}{e^3} \approx 1.34$ IS A LOCAL (& GLOBAL) MAXIMUM.

6. The demand for Barbie with Kung Fu Grip (in thousands) is given as a function of price (in dollars) by $q(p) = 4000 - 30p - 0.1p^2$. For what price will revenue be maximized and what will the maximum revenue be? (Use two different methods to determine your answer.)

change

$00 - 3p + 0.1p^2$

$R = 4000p - 30p^2 - 0.1p^3$

$R'(p) = 4000 - 60p - 0.3p^2$

$0 = \underline{\qquad\qquad}$

$\to p = 52.75$ & -252.75

$\to p = \$52.75$

$q(52.75) = \$112,845$

CALC.