Introduction

To analyze recent trends in spending on Internet advertising and to make reasonable projections, we need a mathematical model of this spending. Where do we start? To apply mathematics to real-world situations like this, we need a good understanding of basic mathematical concepts. Perhaps the most fundamental of these concepts is that of a function: a relationship that shows how one quantity depends on another. Functions may be described numerically and, often, algebraically. They can also be described graphically—a viewpoint that is extremely useful.

The simplest functions—the ones with the simplest formulas and the simplest graphs—are linear functions. Because of their simplicity, they are also among the most useful functions and can often be used to model real-world situations, at least over short periods of time. In discussing linear functions, we will meet the concepts of slope and rate of change, which are the starting point of the mathematics of change.

In the last section of this chapter, we discuss *simple linear regression*: construction of linear functions that best fit given collections of data. Regression is used extensively in applied mathematics, statistics, and quantitative methods in business. The inclusion of regression utilities in computer spreadsheets like Excel® makes this powerful mathematical tool readily available for anyone to use.

algebra Review
For this chapter you should be familiar with real numbers and intervals. To review this material, see Chapter 0.

1.1 Functions from the Numerical and Algebraic Viewpoints

The following table gives the weights of a particular child at various ages in her first year:

Age (months)	0	2	3	4	5	6	9	12
Weight (pounds)	8	9	13	14	16	17	18	19

Let's write $W(0)$ for the child's weight at birth (in pounds), $W(2)$ for her weight at 2 months, and so on (we read $W(0)$ as "W of 0"). Thus, $W(0) = 8$, $W(2) = 9$, $W(3) = 13, \ldots$, $W(12) = 19$. More generally, if we write t for the age of the child (in months) at any time during her first year, then we write $W(t)$ for the weight of the child at age t. We call W a **function** of the variable t, meaning that for each value of t between 0 and 12, W gives us a single corresponding number $W(t)$ (the weight of the child at that age).

In general, we think of a function as a way of producing new objects from old ones. The functions we deal with in this text produce new numbers from old numbers. The numbers we have in mind are the *real* numbers, including not only positive and negative integers and fractions but also numbers like $\sqrt{2}$ or π (see Chapter 0 for more on real numbers). For this reason, the functions we use are called **real-valued functions of a real variable.** For example, the function W takes the child's age in months and returns her weight in pounds at that age (Figure 1).

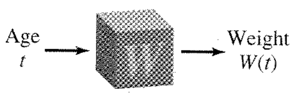


Figure 1

The variable t is called the **independent** variable, while W is called the **variable** as its value depends on t.

A function may be specified in several different ways. It may be specifi **cally,** by giving the values of the function for a number of values of the i variable, as in the preceding table. It may be specified **verbally,** as in "Let I weight of the child at age t months in her first year."[1] In some cases we may use an algebraic formula to calculate the function, and we say that the functi ified **algebraically.** In Section 1.2 we will see that a function may also b **graphically.**

Q: For which values of t does it make sense to ask for W(t)? In other words, for whic the function W defined?

A: Since $W(t)$ refers to the weight of the child at age t months *in her first year*, $W(t$ when t is any number between 0 and 12, that is, when $0 \leq t \leq 12$. Using interval not Appendix A), we can say that $W(t)$ is defined when t is in the interval $[0, 12]$. ■

The set of values of the independent variable for which a function is called its **domain** and is a necessary part of the definition of the function. Not preceding table gives the values of $W(t)$ at only some of the infinitely mai values in the domain $[0, 12]$.

The domain of a function is not always specified explicitly; if no specified for a function f, we take the domain to be the largest set of numbers $f(x)$ makes sense. This "largest possible domain" is sometimes called tl **domain.**

Here is a summary of the terms we've just introduced.

Functions

A **real-valued function f of a real-valued variable x** assigns to each real n a specified set of numbers, called the **domain** of f, a unique real number "f of x." The variable x is called the **independent variable,** and f is called **dent variable.**

quick Examples

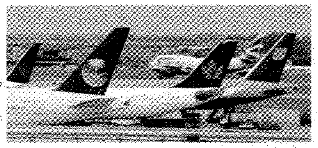
1. Let $W(t)$ be the weight (in pounds) at age t months of a particular child first year. The independent variable is t, and the dependent variable is W, weight. The domain of W is $[0, 12]$ because it was specified that W gives weight during her first year.

2. Let $f(x) = \dfrac{1}{x}$. The function f is specified algebraically. Some specific val

$$f(2) = \frac{1}{2} \qquad f(3) = \frac{1}{3} \qquad f(-1) = \frac{1}{-1} = -1$$

Here, $f(0)$ is not defined because there is no such number as $1/0$. The natu of f consists of all real numbers except zero because $f(x)$ makes sense fo of x other than $x = 0$.

[1] Specifying a function verbally in this way is useful for understanding what the function is doin no numerical information.


Getty Images

Example 1 A Numerically Specified Function: Airline Profits

The following table* shows the cumulative net income of U.S. domestic airlines from January 2000 to the end of year x:

Year x (Since 2000)	0	1	2	3	4
Cumulative Net Income P ($ Billions)	12	2	−34	−51	−61

Viewing P as a function of x, give its domain and the values $P(0)$, $P(2)$, and $P(4)$. Compute $P(3) - P(2)$ and interpret the result. Also estimate and interpret the value $P(3.5)$.

Solution The domain of P is the set of numbers x with $0 \le x \le 4$—that is, [0, 4].

From the table, we have:

$P(0) = 12$ $12 billion net income in 2000

$P(2) = -34$ $34 billion cumulative loss from Jan. 2000 to Dec. 2002

$P(4) = -61$ $61 billion cumulative loss from Jan. 2000 to Dec. 2004

Also,

$$P(3) - P(2) = -51 - (-34) = -17$$

To interpret the result, notice that:

Cumulative net income through 2003 − Cumulative net income through 2002
= Net income in 2003

Thus, the net income in 2003 was −$17 billion. In other words, $17 billion was lost by the airline industry in 2003.

What about $P(3.5)$? Since $P(3) = -51$ and $P(4) = -61$, we estimate that

$P(3.5) \approx -56$ −56 is midway between −51 and −61.

The process of estimating values for a function between points where it is already known is called **interpolation.**

To interpret $P(3.5)$, note that $P(3)$ represents the accumulated net income through 2003, and $P(4)$ represents the accumulated net income through 2004. Thus, $P(3.5)$ represents the accumulated net income through June, 2003.

*2004 figure is an estimate based on first quarter results. SOURCE: Bureau of Transportation Statistics www.bts.gov/ Nov 15 2004.

Before we go on... In Example 1 we should not use the table to estimate $P(x)$ for values of x *outside* the domain—say, for $x = 10$. Estimating values for a function outside a range where it is already known is called **extrapolation.** As a general rule, extrapolation is far less reliable than interpolation: predicting the future from current data is difficult, especially given the vagaries of the marketplace. ■

The two functions we have looked at so far were both specified numerically: we were given numerical values of the function evaluated at *certain* values of the independent variable. It would be more useful if we had a formula that would allow us to

calculate the value of the function for *any* value of the independent variable that is, if the function were specified algebraically.

Example 2 An Algebraically Defined Function

Let f be the function specified by

$$f(x) = x^2 - 25x + 15$$

with domain (−2, 10]. When $0 \le x \le 4$, this formula gives an approxima airline cumulative net income function P in Example 1. Use the formula t $f(0)$, $f(10)$, $f(-1)$, $f(a)$, and $f(x+h)$. Is $f(-2)$ defined?

Solution Let's check first that the values we are asked to calculate are Since the domain is stated to be (−2, 10], the quantities $f(0)$, $f(10)$, and $f($ defined. The quantities $f(a)$ and $f(x+h)$ will also be defined if a and x - derstood to be in (−2, 10]. However, $f(-2)$ is not defined, since −2 is not in (−2, 10].

If we substitute 0 for x in the formula for $f(x)$, we get

$$f(0) = (0)^2 - 25(0) + 15 = 15$$

so $f(0) = 15$. Similarly,

$$f(10) = (10)^2 - 25(10) + 15 = 100 - 250 + 15 = -135$$
$$f(-1) = (-1)^2 - 25(-1) + 15 = 1 + 25 + 15 = 41$$
$$f(a) = a^2 - 25a + 15 \quad \text{Substitute } a \text{ for } x.$$
$$f(x+h) = (x+h)^2 - 25(x+h) + 15 \quad \text{Substitute } (x+h) \text{ for } x.$$
$$= x^2 + 2xh + h^2 - 25x - 25h + 15$$

Note how we placed parentheses around the number at which we are eva function. If we omitted any of these parentheses, we would likely get errors:

$f(-1) = (-1)^2 - 25(-1) + 15$ ✓ NOT $-1^2 - 25(-1) + 15$

$f(x+h) = (x+h)^2 - 25(x+h) + 15$ ✓ NOT $x + h^2 - 25(x+h)$

Note that there is nothing magical about the letter x. We might just as well s

$$f(t) = t^2 - 25t + 15$$

which defines *exactly the same function* as $f(x) = x^2 - 25x + 15$. For e calculate $f(10)$ from the formula for $f(t)$ we would substitute 10 for $f(10) = -135$, just as we did using the formula for $f(x)$.

Before we go on... We said that the function f given in the Example 2 is an tion of the cumulative net income function P of Example 1. The following pares some of their values:[2]

x	0	2	3	4
$P(x)$	12	−34	−51	−61
$f(x)$	15	−31	−51	−69

[2] The function f is a "best-fit," or regression quadratic curve based on the data in Example 1 (c rounded). We will learn more about regression later in this chapter.

We call the algebraic function f an **algebraic model** of U.S. airlines' cumulative net income from Jan. 2000 because it uses an algebraic formula to model—or mathematically represent (approximately)—the cumulative net income. The particular kind of algebraic model we used is called a **quadratic model** (see the end of this section for the names of some commonly used models). ■

Q: The values of f(x) are close to but don't all equal those of P(x). Is this the best we can do with an algebraic model? Can't we get a formula that gives the cumulative net income data exactly?

A: It is possible to find algebraic formulas that give the exact values of $P(x)$, but such formulas would be far more complicated than the one given, and quite possibly less useful.[3] ■

Note **Equation and Function Notation**

Instead of using *function notation*

$$f(x) = x^2 - 25x + 15 \qquad \text{Function notation}$$

we could use equation notation

$$y = x^2 - 25x + 15 \qquad \text{Equation notation}$$

(the choice of the letter y is a convention) and we say that "y is a function of x." When we write a function in this form, the variable x is the independent variable and y is the dependent variable.

We could also write the above function as $f = x^2 - 25x + 15$, in which case the dependent variable would be f.

using *Technology*

Evaluating a function can be tedious to do by hand, but various technologies make this task easier. See the Technology Guides at the end of the chapter to find out how to create a table like the one in Example 3, using a TI-83/84 or Excel. Alternatively, go online and follow:

Chapter 1
→ Tools
→ Function Evaluator & Grapher

to find a utility you can use to evaluate functions like this.

Example 3 Evaluating a Function with Technology

Evaluate the function $f(x) = -0.4x^2 + 7x - 23$ for $x = 0, 1, 2, \ldots, 10$.

Solution

The first couple of evaluations go as follows:

$$f(0) = -0.4(0)^2 + 7(0) - 23 = -23$$
$$f(1) = -0.4(1)^2 + 7(1) - 23 = -0.4 + 7 - 23 = -16.4$$

Note that to evaluate $-0.4x^2$, we first compute x^2 and then multiply by -0.4. Continuing, we get the following table.

x	0	1	2	3	4	5	6	7	8	9	10
$f(x)$	−23	−16.4	−10.6	−5.6	−1.4	2	4.6	6.4	7.4	7.6	7

Sometimes, as in Example 4, we need to use several formulas to specify a single function.

[3] One reason that more complex formulas are often less realistic than simple ones is that it is often random phenomena in the real world, rather than algebraic relationships, that cause data to fluctuate. Attempting to model these random fluctuations using algebraic formulas amounts to imposing mathematical structure where structure does not exist.

using *Technology*

See the Technology Guides at the end of the chapter to see how to evaluate functions like this using a TI-83/84 or Excel. The techniques shown there work for other technologies as well, including the function evaluator that you can find online. Follow:

Chapter 1
→ Tools
→ Function Evaluator & Grapher

Example 4 A Piecewise-Defined Function: EBAY Stock

The price $V(t)$ in dollars of EBAY stock during the 10-week period starting J can be approximated by the following function of time t in weeks ($t = 0$ July 1):*

$$V(t) = \begin{cases} 90 - 4t & \text{if } 0 \le t \le 5 \\ 60 + 2t & \text{if } 5 < t \le 20 \end{cases}$$

What was the approximate price of EBAY stock after 1 week, after 5 week 10 weeks?

Solution

We evaluate the given function at the corresponding value of t:

$t = 1$: $V(1) = 90 - 4(1) = 86$ We use the first formula since $0 \le$

$t = 5$: $V(5) = 90 - 4(5) = 70$ We use the first formula since $0 \le$

$t = 10$: $V(10) = 60 + 2(10) = 80$ We use the second formula since 5

Thus, the price of EBAY stock was \$86 after 1 week, \$70 after 5 weeks, an 10 weeks.

* Source for data: http://money.excite.com, November, 2004

The functions we used in Examples 1–4 above are **mathematical mod** life situations, because they model, or represent, situations in mathematical

Mathematical Modeling

To mathematically model a situation means to represent it in mathematical particular representation used is called a **mathematical model** of the situati matical models do not always represent a situation perfectly or completely. Example 2) represent a situation only approximately, whereas others rep some aspects of the situation.

quick Examples

Situation	*Model*
1. Albano's bank balance is twice Bravo's.	$a = 2b$ (a = Albano's balance, b = Bravo's)
2. The temperature is now 10°F and increasing by 20° per hour.	$T(t) = 10 + 20t$ (t = time in hours, T = temperature)
3. The volume of a rectangular solid with square base is obtained by multiplying the area of its base by its height.	$V = x^2h$ (h = height, x = length of a side of th
4. U.S. airlines' cumulative net income	The table in Example 1 is a **numerica** of U.S. airlines' income. The function in Example 2 is an **algel model** of U.S. airlines' income.
5. EBAY stock price	Example 4 gives a **piecewise algebra** of the EBAY stock price.

Table 1 lists some common types of functions that are often used to model real world situations.

Table 1 Common Types of Algebraic Functions

	Type of Function	*Example*
Linear	$f(x) = mx + b$ m, b constant	$f(x) = 3x - 2$ Technology format: `3*x - 2`
Quadratic	$f(x) = ax^2 + bx + c$ a, b, c constant ($a \neq 0$)	$f(x) = -3x^2 + x - 1$ Technology format: `-3*x^2 + x - 1`
Cubic	$f(x) = ax^3 + bx^2 + cx + d$ a, b, c, d constant ($a \neq 0$)	$f(x) = 2x^3 - 3x^2 + x - 1$ Technology format: `2*x^3 - 3*x^2 + x - 1`
Polynomial	$f(x) = ax^n + bx^{n-1} + \ldots + rx + s$ $a, b, \ldots, r, s$ constant (Includes all of the above functions)	All the above, and $f(x) = x^6 - x^4 + x - 3$ Technology format: `x^6 - x^4 + x - 3`
Exponential	$f(x) = Ab^x$ A, b constant (b positive)	$f(x) = 3(2^x)$ Technology format: `3*2^x`
Rational	$f(x) = \dfrac{P(x)}{Q(x)}$ $P(x)$ and $Q(x)$ polynomials	$f(x) = \dfrac{x^2 - 1}{2x + 5}$ Technology format: `(x^2 - 1)/(2*x + 5)`

Functions and models other than linear ones are called **nonlinear.**

1.1 EXERCISES

⊛ denotes basic skills exercises

tech Ex indicates exercises that should be solved using technology

In Exercises 1–4, evaluate or estimate each expression based on the following table. hint [see Example 1]

x	−3	−2	−1	0	1	2	3
$f(x)$	1	2	4	2	1	0.5	0.25

1. ⊛ **a.** $f(0)$ **b.** $f(2)$

2. ⊛ **a.** $f(-1)$ **b.** $f(1)$

3. ⊛ **a.** $f(2) - f(-2)$ **b.** $f(-1)f(-2)$ **c.** $-2f(-1)$

4. ⊛ **a.** $f(1) - f(-1)$ **b.** $f(1)f(-2)$ **c.** $3f(-2)$

5. ⊛ Given $f(x) = 4x - 3$, find **a.** $f(-1)$ **b.** $f(0)$ **c.** $f(1)$ **d.** $f(y)$ **e.** $f(a + b)$ hint [see Example 2]

6. ⊛ Given $f(x) = -3x + 4$, find **a.** $f(-1)$ **b.** $f(0)$ **c.** $f(1)$ **d.** $f(y)$ **e.** $f(a + b)$

7. ⊛ Given $f(x) = x^2 + 2x + 3$, find **a.** $f(0)$ **b.** $f(1)$ **c.** $f(-1)$ **d.** $f(-3)$ **e.** $f(a)$ **f.** $f(x + h)$

8. ⊛ Given $g(x) = 2x^2 - x + 1$, find **a.** $g(0)$ **b.** $g(-1)$ **c.** $g(r)$ **d.** $g(x + h)$

9. ⊛ Given $g(s) = s^2 + \dfrac{1}{s}$, find **a.** $g(1)$ **b.** $g(-1)$ **c.** $g(4)$ **d.** $g(x)$ **e.** $g(s + h)$ **f.** $g(s + h) - g(s)$

10. ⊛ Given $h(r) = \dfrac{1}{r + 4}$, find **a.** $h(0)$ **b.** $h(-3)$ **c.** $h(-5)$ **d.** $h(x^2)$ **e.** $h(x^2 + 1)$ **f.** $h(x^2) + 1$

11. ⊛ Given $f(t) = \begin{cases} -t & \text{if } t < 0 \\ t^2 & \text{if } 0 \le t < 4 \\ t & \text{if } t \ge 4 \end{cases}$

find **a.** $f(-1)$ **b.** $f(1)$ **c.** $f(4) - f(2)$ **d.** $f(3)f(-3)$ hint [see Example 4]

12. ⊛ Given $f(t) = \begin{cases} t - 1 & \text{if } t \le 1 \\ 2t & \text{if } 1 < t < 5 \\ t^3 & \text{if } t \ge 5 \end{cases}$

find **a.** $f(0)$ **b.** $f(1)$ **c.** $f(4) - f(2)$ **d.** $f(5) + f(-5)$

In Exercises 13–16, say whether $f(x)$ is defined for the given values of x. If it is defined, give its value.

13. ⊛ $f(x) = x - \dfrac{1}{x^2}$, with domain $(0, +\infty)$ **a.** $x = 4$ **b.** $x = 0$ **c.** $x = -1$

14. ⊛ $f(x) = \dfrac{2}{x} - x^2$, with domain $[2, +\infty)$ **a.** $x = 4$ **b.** $x = 0$ **c.** $x = 1$

15. ⊛ $f(x) = \sqrt{x + 10}$, with domain $[-10, 0)$ **a.** $x = 0$ **b.** $x = 9$ **c.** $x = -10$

16. ⊛ $f(x) = \sqrt{9 - x^2}$, with domain $(-3, 3)$ **a.** $x = 0$ **b.** $x = 3$ **c.** $x = -3$

*In Exercises 17–20, find and simplify **(a)** $f(x + h) - f(x)$ **(b)** $\dfrac{f(x + h) - f(x)}{h}$*

17. $f(x) = x^2$

18. $f(x) = 3x - 1$

19. $f(x) = 2 - x^2$

20. $f(x) = x^2 + x$

In Exercises 21–24, first give the technology formula for the given function and then use technology to evaluate the function for the given values of x (when defined there).

21. tech Ex $f(x) = 0.1x^2 - 4x + 5$; $x = 0, 1, \ldots, 10$

22. tech Ex $g(x) = 0.4x^2 - 6x - 0.1$; $x = -5, -4, \ldots, 4, 5$

23. tech Ex $h(x) = \dfrac{x^2 - 1}{x^2 + 1}$; $x = 0.5, 1.5, 2.5, \ldots, 10.5$ (Round all answers to four decimal places.)

24. tech Ex $r(x) = \dfrac{2x^2 + 1}{2x^2 - 1}$; $x = -1, 0, 1, \ldots, 9$ (Round all answers to four decimal places.)

Applications

25. ⊛ ***Employment*** The following table lists the approximate number of people employed in the U.S. during the period 1995–2001, on July 1 of each year[4] ($t = 5$ represents 1995):

Year t	5	6	7	8	9	10	11
Employment $P(t)$ (Millions)	117	120	123	125	130	132	132

a. Find or estimate $P(5)$, $P(10)$, and $P(9.5)$. Interpret your answers.

b. What is the domain of P?

[4] The given values represent nonfarm employment, and are approximate. SOURCE: Bureau of Labor Statistics/*The New York Times,* December 17, 2001, p. C3.

26. ⊛ ***Cell Phone Sales*** The following table lis (after-tax revenue) at the Finnish cell phone for each year in the period 1995–2001[5] (t 1995):

Year t	5	6	7	8
Nokia Net Sales $P(t)$ (Billions of Dollars)	8	8	10	16

a. Find or estimate $P(5)$, $P(10)$, and $P(7.5)$. Ir answers.

b. What is the domain of P?

27. ⊛ ***Trade with China*** The value of U.S. trade v 1994 through 2004 can be approximated by

$$C(t) = 3t^2 - 7t + 50 \text{ billion dol}$$

(t is time in years since 1994).[6]

a. Find an appropriate domain of C. Is $t \ge 0$ a domain? Why or why not?

b. Compute $C(10)$. What does the answer say with China?

28. ⊛ ***Scientific Research*** The number of rese *Physics Review* that were written by researcl from 1983 through 2003 can be approximatec

$$A(t) = -0.01t^2 + 0.24t + 3.4 \text{ hundre}$$

(t is time in years since 1983).[7]

a. Find an appropriate domain of A. Is $t \le 20$ domain? Why or why not?

b. Compute $A(10)$. What does the answer say number of research articles?

29. ⊛ ***Spending on Corrections in the 90s*** The shows the annual spending by all states in the tions ($t = 0$ represents the year 1990):[8]

Year (t)	0	2	4	(
Spending (\$ billion)	16	18	22	2{

a. Which of the following functions best fits tl (*Warning*: none of them fits exactly, but onc closely than the others.)

(1) $S(t) = -0.2t^2 + t + 16$

(2) $S(t) = 0.2t^2 + t + 16$

(3) $S(t) = t + 16$

[5] SOURCE: Nokia/*New York Times,* February 6, 2002, p. A

[6] Based on a regression by the authors. Source for d Bureau/*New York Times,* September 23, 2004, p. C1.

[7] Based on a regression by the authors. Source for da Physical Society/*New York Times,* May 3, 2003, p. A1.

[8] Data are rounded. SOURCE: National Association of Statc *The New York Times,* February 28, 1999, p. A1.

b. Use your answer to part (a) to "predict" spending on corrections in 1998, assuming that the trend continued.

30. ⊛ ***Spending on Corrections in the 90s*** Repeat Exercise 29, this time choosing from the following functions:

(1) $S(t) = 16 + 2t$

(2) $S(t) = 16 + t + 0.5t^2$

(3) $S(t) = 16 + t - 0.5t^2$

31. ***Demand*** The demand for Sigma Mu Fraternity plastic brownie dishes is

$$q(p) = 361{,}201 - (p + 1)^2$$

where q represents the number of brownie dishes Sigma Mu can sell each month at a price of p. Use this function to determine

a. The number of brownie dishes Sigma Mu can sell each month if the price is set at 50¢,

b. The number of brownie dishes they can unload each month if they give them away,

c. The lowest price at which Sigma Mu will be unable to sell any dishes.

32. ***Revenue*** The total weekly revenue earned at Royal Ruby Retailers is given by

$$R(p) = -\frac{4}{3}p^2 + 80p$$

where p is the price (in dollars) RRR charges per ruby. Use this function to determine

a. The weekly revenue, to the nearest dollar, when the price is set at \$20/ruby,

b. The weekly revenue, to the nearest dollar, when the price is set at \$200/ruby (interpret your result).

c. The price RRR should charge in order to obtain a weekly revenue of \$1200.

33. ⊛ ***Processor Speeds*** The processor speed, in megahertz, of Intel processors could be approximated by the following function of time t in years since the start of 1995:[9]

$$P(t) = \begin{cases} 75t + 200 & \text{if } 0 \le t \le 4 \\ 600t - 1900 & \text{if } 4 < t \le 9 \end{cases}$$

a. Evaluate $P(0)$, $P(4)$, and $P(5)$ and interpret the results.

b. Use the model to estimate when processor speeds first hit 2.0 gigahertz (1 gigahertz = 1000 megahertz).

c. techEx Use technology to generate a table of values for $P(t)$ with $t = 0, 1, \ldots, 9$.

34. ⊛ ***Leading Economic Indicators*** The value of the Conference Board Index of 10 economic indicators in the U.S. could be approximated by the following function of time t in months since the end of December, 2002:[10]

$$E(t) = \begin{cases} 0.4t + 110 & \text{if } 6 \le t \le 15 \\ -0.2t + 119 & \text{if } 15 < t \le 20 \end{cases}$$

a. Estimate $E(10)$, $E(15)$, and $E(20)$ and interpret the results.

b. Use the model to estimate when—prior to March, 2004—the index was 115.

c. techEx Use technology to generate a table of values for $E(t)$ with $t = 6, 7, \ldots, 20$.

35. techEx ***Television Advertising*** The cost, in millions of dollars, of a 30-second television ad during the Super Bowl from 1990 to 2001 can be approximated by the following piecewise linear function ($t = 0$ represents 1990):[11]

$$C(t) = \begin{cases} 0.08t + 0.6 & \text{if } 0 \le t < 8 \\ 0.355t - 1.6 & \text{if } 8 \le t \le 11 \end{cases}$$

a. Give the technology formula for C and complete the following table of values of the function C.

t	0	1	2	3	4	5	6	7	8	9	10	11
$C(t)$												

b. Between 1998 and 2000, the cost of a Super Bowl ad was increasing at a rate of \$____ million per year

36. techEx ***Internet Purchases*** The percentage $p(t)$ of new car buyers who used the Internet for research or purchase since 1997 is given by the following function[12] ($t = 0$ represents 1997):

$$p(t) = \begin{cases} 10t + 15 & \text{if } 0 \le t < 1 \\ 15t + 10 & \text{if } 1 \le t \le 4 \end{cases}$$

a. Give the technology formula for p and complete the following table of values of the function p.

t	0	0.5	1	1.5	2	2.5	3	3.5	4
$p(t)$									

b. Between 1998 and 2000, the percentage of buyers of new cars who used the Internet for research or purchase was increasing at a rate of ____% per year

37. ***Income Taxes*** The U.S. Federal income tax is a function of taxable income. Write T for the tax owed on a taxable income of I dollars. For tax year 2005, the function T for a single taxpayer was specified as follows:

If your taxable income was Over—	*But not over—*	*Your tax is*	*of the amount over—*
\$0	7,300	.... **10%**	**\$0**
7,300	29,700	**\$730.00 + 15%**	**7,300**
29,700	71,950	**4,090.00 + 25%**	**29,700**
71,950	150,150	**14,652.50 + 28%**	**71,950**
150,150	326,450	**36,548,50 + 33%**	**150,150**
326,450	....	**94,727,50 + 35%**	**326,450**

What was the tax owed by a single taxpayer on a taxable income of \$26,000? On a taxable income of \$65,000?

38. ***Income Taxes*** The income tax function T in Exercise 37 can also be written in the following form:

$$T(I) = \begin{cases} 0.10I & \text{if } 0 < I \le 7{,}300 \\ 730 + 0.15(I - 7{,}300) & \text{if } 7{,}300 < I \le 29{,}700 \\ 4{,}090.00 + 0.25(I - 29{,}700) & \text{if } 29{,}700 < I \le 71{,}950 \\ 14{,}652.50 + 0.28(I - 71{,}950) & \text{if } 71{,}950 < I \le 150{,}150 \\ 36{,}548.50 + 0.33(I - 150{,}150) & \text{if } 150{,}150 < I \le 326{,}450 \\ 94{,}727.50 + 0.35(I - 326{,}450) & \text{if } I > 326{,}450 \end{cases}$$

What was the tax owed by a single taxpayer on a taxable income of \$25,000? On a taxable income of \$125,000?

39. ***Toxic Waste Treatment*** The cost of treating waste by removing PCPs goes up rapidly as the quantity of PCPs removed goes up. Here is a possible model:

$$C(q) = 2000 + 100q^2$$

where q is the reduction in toxicity (in pounds of PCPs removed per day) and $C(q)$ is the daily cost (in dollars) of this reduction.

a. Find the cost of removing 10 pounds of PCPs per day.

b. Government subsidies for toxic waste cleanup amount to

$$S(q) = 500q$$

where q is as above and $S(q)$ is the daily dollar subsidy. Calculate the net cost function $N(q)$ (the cost of removing q pounds of PCPs per day after the subsidy is taken into account), given the cost function and subsidy above, and find the net cost of removing 20 pounds of PCPs per day.

40. ***Dental Plans*** A company pays for its employees' dental coverage at an annual cost C given by

$$C(q) = 1000 + 100\sqrt{q}$$

where q is the number of employees covered and $C(q)$ is the annual cost in dollars.

a. If the company has 100 employees, find its annual outlay for dental coverage.

b. Assuming that the government subsidizes cove annual dollar amount of

$$S(q) = 200q$$

calculate the net cost function $N(q)$ to the com calculate the net cost of subsidizing its 100 em Comment on your answer.

41. techEx ***Acquisition of Language*** The percen children who can speak at least single words by months can be approximated by the equation[13]

$$p(t) = 100\left(1 - \frac{12{,}200}{t^{4.48}}\right) \quad (t \ge 8.5)$$

a. Give a technology formula for p.

b. Create a table of values of p for $t = 9, 10, \ldots$ (rounding answers to one decimal place).

c. What percentage of children can speak at least words by the age of 12 months?

d. By what age are 90% or more children speakir single words?

42. techEx ***Acquisition of Language*** The percen children who can speak in sentences of five or m the age of t months can be approximated by th

$$p(t) = 100\left(1 - \frac{5.27 \times 10^{17}}{t^{12}}\right) \quad (t \ge 30$$

a. Give a technology formula for p.

b. Create a table of values of p for $t = 30, 31, \ldots$ (rounding answers to one decimal place).

c. What percentage of children can speak in sent or more words by the age of 36 months?

d. By what age are 75% or more children speakir tences of five or more words?

Communication and Reasoning Exercises

43. ⊛ If the market price m of gold varies with time t, dependent variable is ___ and the dependent var

44. ⊛ Complete the following sentence: If weekly pro ified as a function of selling price s, then the indep able is ___ and the dependent variable is ___.

45. ⊛ Complete the following: The function notation tion $y = 4x^2 - 2$ is ____.

46. ⊛ Complete the following: The equation notatior $-0.34t^2 + 0.1t$ is ____.

[9] Source: Sandpile.org/*New York Times*, May 17, 2004, p. C1.

[10] Source: The Conference Board/*New York Times*, November 19, 2004, p. C7.

[11] Source: *New York Times*, January 26, 2001, p. C1.

[12] Model is based on data through 2000 (the 2000 value is estimated). Source: J. D. Power Associates/*The New York Times*, January 25, 2000, p. C1.

[13] The model is the authors' and is based on data presented *The Emergence of Intelligence* by William H. Calvin, *Scient* October, 1994, pp. 101–107.

[14] Ibid.

⊛ basic skills techEx technology exercise

47. You now have 200 sound files on your hard drive, and this number is increasing by 10 sound files each day. Find a mathematical model for this situation.

48. The amount of free space left on your hard drive is now 50 gigabytes (GB) and is decreasing by 5 GB/month. Find a mathematical model for this situation.

49. Why is the following assertion false? "If $f(x) = x^2 - 1$, then $f(x + h) = x^2 + h - 1$."

50. Why is the following assertion false? "If $f(2) = 2$ and $f(4) = 4$, then $f(3) = 3$."

51. True or false: Every function can be specified numerically.

52. Which supplies more information about a situation: a numerical model or an algebraic model?

● basic skills tech Ex technology exercise

1.2 Functions from the Graphical Viewpoint

Consider again the function W discussed in Section 1.1, giving a child's weight during her first year. If we represent the data given in Section 1.1 graphically by plotting the given pairs of numbers $(t, W(t))$, we get Figure 2. (We have connected successive points by line segments.)

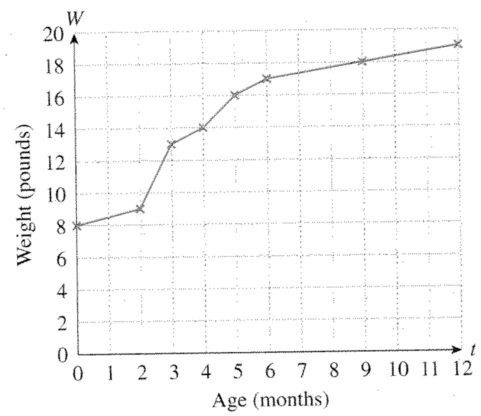


Figure 2

Suppose now that we had only the graph without the table of data given in Section 1.1. We could use the graph to find values of W. For instance, to find $W(9)$ from the graph we do the following:

1. Find the desired value of t at the bottom of the graph ($t = 9$ in this case).
2. Estimate the height (W-coordinate) of the corresponding point on the graph (18 in this case).

Thus, $W(9) \approx 18$ pounds.[15]

We say that Figure 2 specifies the function W **graphically.** The graph is not a very accurate specification of W; the actual weight of the child would follow a smooth curve rather than a jagged line. However, the jagged line is useful in that it permit polate: for instance, we can estimate that $W(1) \approx 8.5$ pounds.

[15] In a graphically defined function, we can never know the y-coordinates of points exactly; no matter how accurately a graph is drawn, we can only obtain *approximate* values of the coordinates of points. That is why we have been using the word *estimate* rather than *calculate* and why we say "$W(9) \approx 18$" rather than "$W(9) = 18$."

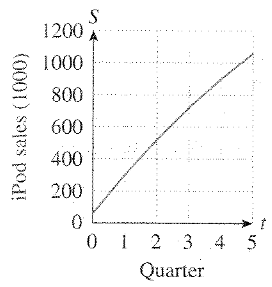


Figure 3

Bartomeu Amengual/Index Stock Imagery

Example 1 A Function Specified Graphically: iPod Sales

Figure 3 shows the approximate quarterly sales of iPods for the second qua through the third quarter in 2004 ($t = 0$ represents the second quarter of 20 Estimate and interpret $S(1)$, $S(4)$, and $S(5)$. What is the domain of S?

Solution We carefully estimate the S-coordinates of the points with t-coor and 5.

$$S(1) \approx 300$$

meaning that iPod sales in the third quarter of 2003 ($t = 1$) were approxima units.

$$S(4) \approx 900$$

meaning that iPod sales in the second quarter of 2004 ($t = 4$) were ap 900,000 units.

$$S(5) \approx 1050$$

meaning that iPod sales in the third quarter of 2004 ($t = 5$) were ap 1,050,000 units.

The domain of S is the set of all values of t for which $S(t)$ is defined: or [0, 5].

* Accurate sales figures are available from Apple financial statements, www.apple.com

Sometimes we are interested in drawing the graph of a function that has be in some other way—perhaps numerically or algebraically. We do this by plo with coordinates $(x, f(x))$.[16] Here is the formal definition of a graph.

Graph of a Function

The **graph of the function f** is the set of all points $(x, f(x))$ in the xy plan *restrict the values of x to lie in the domain of f.*

quick Example

To sketch the graph of the function

$$f(x) = x^2 \quad \text{Function notation}$$

or

$$y = x^2 \quad \text{Equation notation}$$

with domain the set of all real numbers, first choose some convenient values domain and compute the corresponding y-coordinates.

[16] Graphing utilities typically draw graphs by plotting and connecting a large number of points.