47. You now have 200 sound files on your hard drive, and this number is increasing by 10 sound files each day. Find a mathematical model for this situation.

48. The amount of free space left on your hard drive is now 50 gigabytes (GB) and is decreasing by 5 GB/month. Find a mathematical model for this situation.

49. Why is the following assertion false? "If $f(x) = x^2 - 1$, then $f(x+h) = x^2 + h - 1$."

50. Why is the following assertion false? "If $f(2) = 2$ and $f(4) = 4$, then $f(3) = 3$."

51. True or false: Every function can be specified numerically.

52. Which supplies more information about a situation: a numerical model or an algebraic model?

● basic skills   tech Ex technology exercise

1.2 Functions from the Graphical Viewpoint

Consider again the function W discussed in Section 1.1, giving a child's weight during her first year. If we represent the data given in Section 1.1 graphically by plotting the given pairs of numbers $(t, W(t))$, we get Figure 2. (We have connected successive points by line segments.)

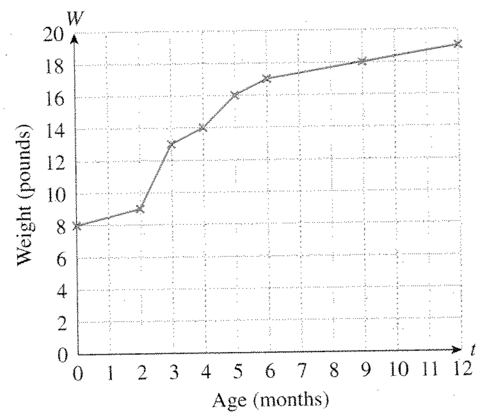


Figure 2

Suppose now that we had only the graph without the table of data given in Section 1.1. We could use the graph to find values of W. For instance, to find $W(9)$ from the graph we do the following:

1. Find the desired value of t at the bottom of the graph ($t = 9$ in this case).
2. Estimate the height (W-coordinate) of the corresponding point on the graph (18 in this case).

Thus, $W(9) \approx 18$ pounds.[15]

We say that Figure 2 specifies the function W **graphically.** The graph is not a very accurate specification of W; the actual weight of the child would follow a smooth curve

[15] In a graphically defined function, we can never know the y-coordinates of points exactly; no matter how accurately a graph is drawn, we can only obtain *approximate* values of the coordinates of points. That is why we have been using the word *estimate* rather than *calculate* and why we say "$W(9) \approx 18$" rather than "$W(9) = 18$."

rather than a jagged line. However, the jagged line is useful in that it permit polate: for instance, we can estimate that $W(1) \approx 8.5$ pounds.

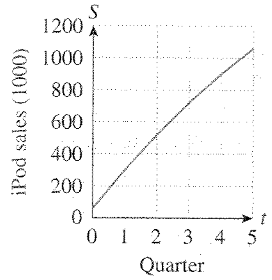
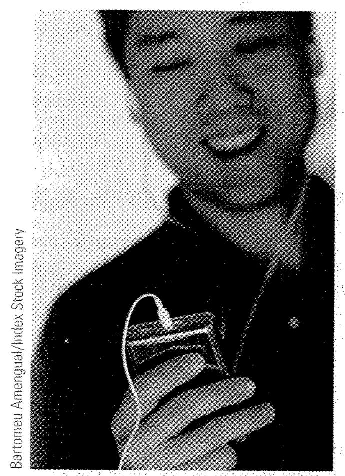


Figure 3



Example 1 A Function Specified Graphically: iPod Sales

Figure 3 shows the approximate quarterly sales of iPods for the second qua through the third quarter in 2004 ($t = 0$ represents the second quarter of 20 Estimate and interpret $S(1)$, $S(4)$, and $S(5)$. What is the domain of S?

Solution We carefully estimate the S-coordinates of the points with t-coor and 5.

$$S(1) \approx 300$$

meaning that iPod sales in the third quarter of 2003 ($t = 1$) were approximat units.

$$S(4) \approx 900$$

meaning that iPod sales in the second quarter of 2004 ($t = 4$) were ap 900,000 units.

$$S(5) \approx 1050$$

meaning that iPod sales in the third quarter of 2004 ($t = 5$) were ap 1,050,000 units.

The domain of S is the set of all values of t for which $S(t)$ is defined: or [0, 5].

* Accurate sales figures are available from Apple financial statements, www.apple.com

Sometimes we are interested in drawing the graph of a function that has be in some other way—perhaps numerically or algebraically. We do this by plo with coordinates $(x, f(x))$.[16] Here is the formal definition of a graph.

Graph of a Function

The **graph of the function f** is the set of all points $(x, f(x))$ in the xy plan *restrict the values of x to lie in the domain of f.*

quick Example To sketch the graph of the function

$$f(x) = x^2 \quad \text{Function notation}$$

or

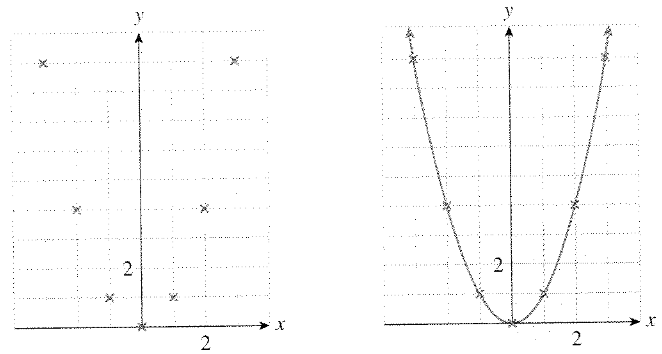
$$y = x^2 \quad \text{Equation notation}$$

with domain the set of all real numbers, first choose some convenient values domain and compute the corresponding y-coordinates.

[16] Graphing utilities typically draw graphs by plotting and connecting a large number of points.

x	−3	−2	−1	0	1	2	3
$y = x^2$	9	4	1	0	1	4	9

Plotting these points gives the picture on the left, suggesting the graph on the right.*



(This particular curve happens to be called a **parabola,** and its lowest point, at the origin, is called its **vertex.**)

* If you plot more points, you will find that they lie on a smooth curve as shown. That is why we did not use line segments to connect the points.

To draw the graph of a function, we often do as we did in the Quick Example above: We plot points of the form $(x, f(x))$ for several values of x in the domain of f, until we can get a good idea of the shape of the entire graph. (Calculus can give us information that allows us to draw a graph with relatively few points plotted.)

Example 2 Drawing the Graph of a Function: Web-Site Revenue

The monthly revenue† R from users logging on to your gaming site depends on the monthly access fee p you charge according to the formula

$$R(p) = -5600p^2 + 14{,}000p \qquad (0 \le p \le 2.5)$$

(R and p are in dollars.) Sketch the graph of R. Find the access fee that will result in the largest monthly revenue.

† The **revenue** resulting from one or more business transactions is the total payment received, sometimes called the gross proceeds.

using Technology

As the name suggests, graphing calculators are designed for graphing functions. Excel is also very good at drawing graphs. See the Technology Guides at the end of the chapter to find out how to graph functions using a TI-83/84 or Excel. Alternatively, there are several graphers available online.

Follow:

Chapter 1
→ Tools

and then click on any of the following:

Function Evaluator & Grapher	Gives a small graph and also values
Java Graphing Utility	A high-quality Java grapher
Excel Graphing Utility	An Excel sheet that graphs*

*Since the Excel grapher requires macros, make sure that macros are *enabled* when Excel prompts you. If macros are disabled, the grapher will not work.

Solution To sketch the graph of R by hand, we plot points of the form (several values of p in the domain [0, 2.5] of R. First, we calculate several p

p	0	0.5	1	1.5	2
$R(p) = -5600p^2 + 14{,}000p$	0	5600	8400	8400	5600

Graphing these points gives the graph shown in Figure 4(a), suggesting shown in Figure 4(b).

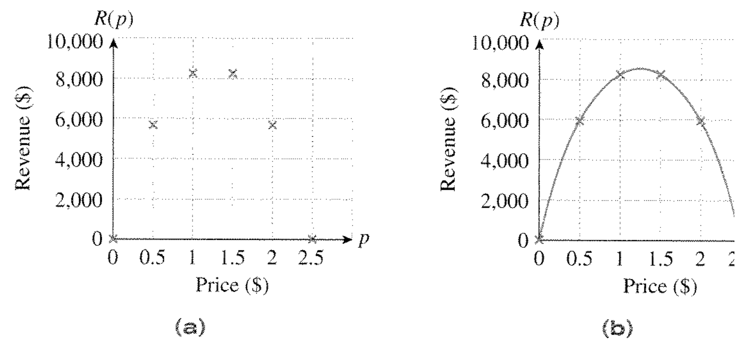


Figure 4

The revenue graph appears to reach its highest point when $p = 1.25$, so s cess fee at $1.25 appears to result in the largest monthly revenue.*

* We are hedging our language with words like *suggesting* and *appears* because the few poin ted don't, by themselves, allow us to draw these conclusions with certainty.

Note **Switching Between Equation and Function Notation**

As we discussed after Example 2 in Section 1.1, we can write the functior ple 2 above in equation notation as

$$R = -5600p^2 + 14{,}000p \qquad \text{Equation notation}$$

The independent variable is p, and the dependent variable is R. Function equation notation, using the same letter for the function name and the dep able, are often used interchangeably. It is important to be able to switch ba easily from function notation to equation notation.

Vertical Line Test

Every point in the graph of a function has the form $(x, f(x))$ for some x ir of f. Since f assigns a *single* value $f(x)$ to each value of x in the domain, it in the graph of f, there should be only one y corresponding to any such namely, $y = f(x)$. In other words, *the graph of a function cannot contain points with the same x-coordinate—that is, two or more points on the same*

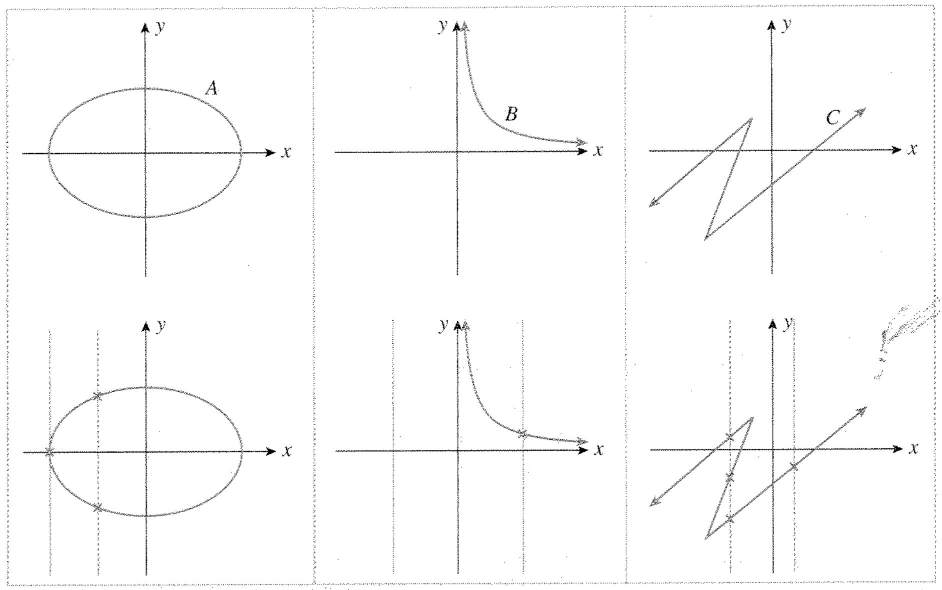
On the other hand, a vertical line at a value of x not in the domain will not contain any points in the graph. This gives us the following rule:

Vertical-Line Test

For a graph to be the graph of a function, every vertical line must intersect the graph in *at most* one point.

quick Examples

As illustrated below, only graph B passes the vertical line test, so only graph B is the graph of a function.



Graphing Piecewise-Defined Functions

Let us revisit the EBAY stock example from Section 1.1.

using *Technology*

To graph the function V using technology, consult the Technology Guides for Example 4 of Section 1.1 to see how to enter this piecewise-defined function. The Technology Guides for Example 2 of this section show how to then draw the graph.

Example 3 Graphing a Piecewise-Defined Function: EBAY Stock

The price $V(t)$ in dollars of EBAY stock during the 10-week period starting July 1, 2004 can be approximated by the following function of time t in weeks ($t = 0$ represents July 1):*

$$V(t) = \begin{cases} 90 - 4t & \text{if } 0 \le t \le 5 \\ 60 + 2t & \text{if } 5 < t \le 20 \end{cases}$$

Graph the function V.

Solution As in Example 2, we can sketch the graph of V by hand by computing $V(t)$ for a number of values of t, plotting these points on the graph, and then connecting them.

* Source for data: http://money.excite.com, November, 2004

t	0	5	10	15	20
$V(t)$	90	70	80	90	100

First formula Second formula

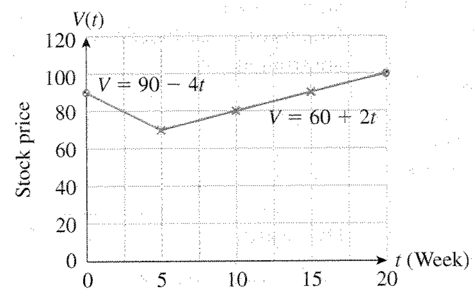


Figure 5

The graph (Figure 5) has the following features:

1. The first formula (the descending line) is used for $0 \le t \le 5$.
2. The second formula (the ascending line) is used for $5 < t \le 20$.
3. The domain is [0, 20], so the graph is cut off at $t = 0$ and $t = 20$.
4. The heavy dots at the ends indicate the endpoints of the domain.

using *Technology*

See the Technology Guide at the end of the chapter for comments on graphing this function using a TI-83/84 or Excel. The formula with inequalities used to graph this function in Excel also works on the various graphers online.

Example 4 Graphing More Complicated Piecewise-Define Functions

Graph the function f specified by

$$f(x) = \begin{cases} -1 & \text{if } -4 \le x < -1 \\ x & \text{if } -1 \le x \le 1 \\ x^2 - 1 & \text{if } 1 < x \le 2 \end{cases}$$

Solution The domain of f is [−4, 2], since $f(x)$ is only specified when
Further, the function changes formulas when $x = -1$ and $x = 1$.

To sketch the graph by hand, we first sketch the three graphs $y = -1$, $y = x$, and $y = x^2 - 1$, and then use the appropriate portion of each (Figure 6).

Note that solid dots indicate points on the graph, whereas the open dots indicate points *not* on the graph. For example, when $x = 1$, the inequalities in the formula tell us that we are to use the middle formula (x) rather than the bottom one ($x^2 - 1$). Thus, $f(1) = 1$, not 0, so we place a solid dot at (1, 1) and an open dot at (1, 0).

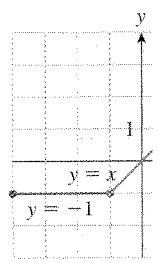


Figure 6

We end this section with a list of some useful types of functions and (Table 2).

New Functions from Old: Scaled and Shifted Functions (Optional Section)

Online, follow:

Chapter 1

→ Online Text

→ New Functions From Old: Scaled and Shifted Functions

where you will find complete interactive text, examples, and exercises on scaling and translating the graph of a function by changing the formula.

Table 2 Functions and Their Graphs

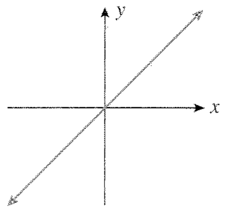
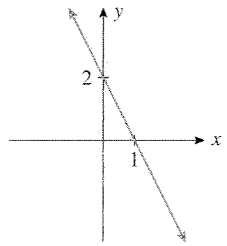
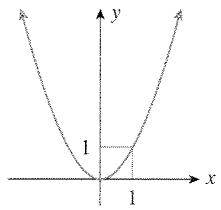
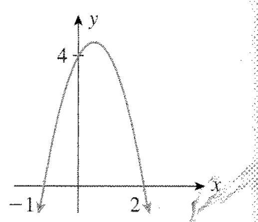
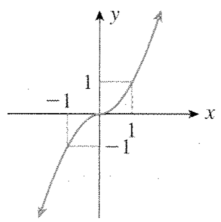
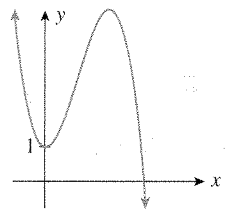
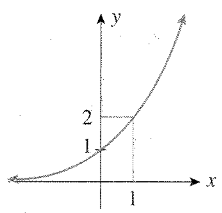
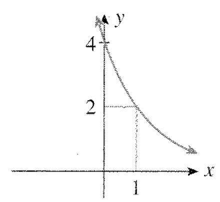
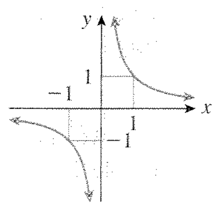
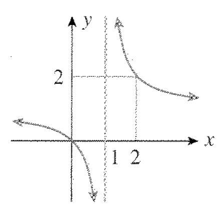
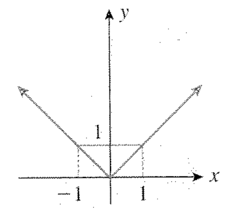
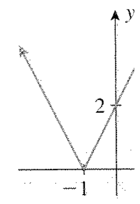
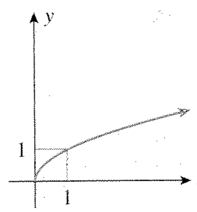
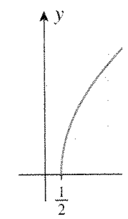
Type of Function	*Examples*	
Linear $f(x) = mx + b$ m, b constant Graphs of linear functions are straight lines.	$y = x$ 	$y = -2x + 2$ 
Quadratic $f(x) = ax^2 + bx + c$ a, b, c constant ($a \neq 0$) Graphs of quadratic functions are called **parabolas.**	$y = x^2$ 	$y = -2x^2 + 2x + 4$ 
Technology formulas:	x^2	-2*x^2 + 2*x + 4
Cubic $f(x) = ax^3 + bx^2 + cx + d$ a, b, c, d constant ($a \neq 0$)	$y = x^3$ 	$y = -x^3 + 3x^2 + 1$ 
Technology formulas:	x^3	-x^3 + 3*x^2 + 1
Exponential $f(x) = Ab^x$ A, b constant ($b > 0$ and $b \neq 1$)	$y = 2^x$ 	$y = 4(0.5)^x$ 
Technology formulas:	2^x	4*0.5^x
Rational $f(x) = \frac{P(x)}{Q(x)}$; $P(x)$ and $Q(x)$ polynomials The graph of $y = 1/x$ is a **hyperbola.** The domain excludes zero since $1/0$ is not defined.	$y = \frac{1}{x}$ 	$y = \frac{x}{x-1}$ 
Technology formulas:	1/x	x/(x - 1)

Table 2 *(Continued)*

Type of Function	*Examples*	
Absolute value For x positive or zero, the graph of $y = \|x\|$ is the same as that of $y = x$. For x negative or zero, it is the same as that of $y = -x$.	$y = \|x\|$ 	$y = \|2x$ 
Technology formulas:	abs(x)	abs(2*
Square Root The domain of $y = \sqrt{x}$ must be restricted to the nonnegative numbers, since the square root of a negative number is not real. Its graph is the top half of a horizontally oriented parabola.	$y = \sqrt{x}$ 	$y = \sqrt{4}$ 
Technology Formulas:	x^0.5 or √(x)	(4*x-2) √(4*x-2)

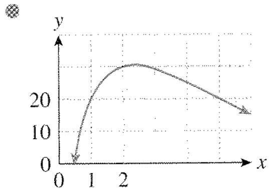
1.2 EXERCISES

⊛ denotes basic skills exercises

tech Ex indicates exercises that should be solved using technology

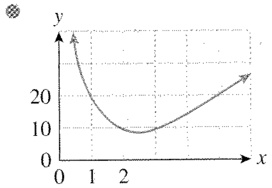
In Exercises 1–4, use the graph of the function f to find approximations of the given values. hint [see Example 1]

1. ⊛



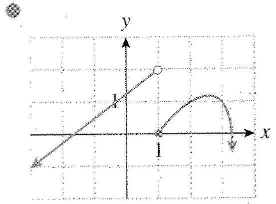
a. $f(1)$
b. $f(2)$
c. $f(3)$
d. $f(5)$
e. $f(3) - f(2)$

2. ⊛



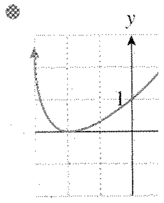
a. $f(1)$
b. $f(2)$
c. $f(3)$
d. $f(5)$
e. $f(3) - f(2)$

3. ⊛



a. $f(-3)$ **b.** $f(0)$
c. $f(1)$ **d.** $f(2)$
e. $\frac{f(3) - f(2)}{3 - 2}$

4. ⊛



a. $f(-2)$
c. $f(1)$
e. $\frac{f(3) - f(}{3 - 1}$

In Exercises 5 and 6, match the functions to the grap technology to draw the graphs is suggested, but not

5. tech Ex

a. $f(x) = x \quad (-1 \le x \le 1)$
b. $f(x) = -x \quad (-1 \le x \le 1)$

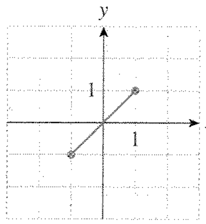
c. $f(x) = \sqrt{x}$ $(0 < x < 4)$

d. $f(x) = x + \frac{1}{x} - 2$ $(0 < x < 4)$

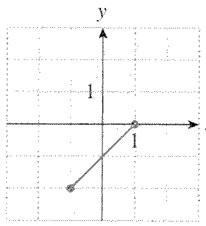
e. $f(x) = |x|$ $(-1 \le x \le 1)$

f. $f(x) = x - 1$ $(-1 \le x \le 1)$

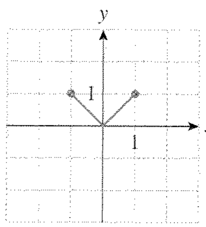
(I)



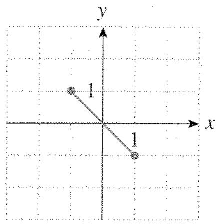
(II)



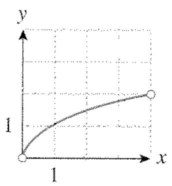
(III)



(IV)



(V)



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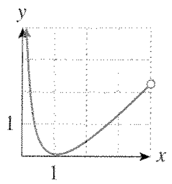


6. tech Ex

a. $f(x) = -x + 4$ $(0 < x \le 4)$

b. $f(x) = 2 - |x|$ $(-2 < x \le 2)$

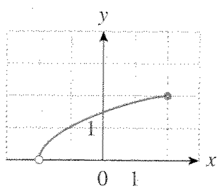
c. $f(x) = \sqrt{x + 2}$ $(-2 < x \le 2)$

d. $f(x) = -x^2 + 2$ $(-2 < x \le 2)$

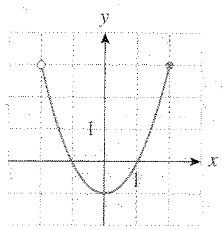
e. $f(x) = \frac{1}{x} - 1$ $(0 < x \le 4)$

f. $f(x) = x^2 - 1$ $(-2 < x \le 2)$

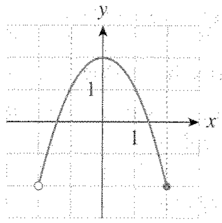
(I)



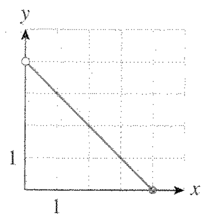
(II)



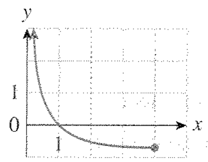
(III)



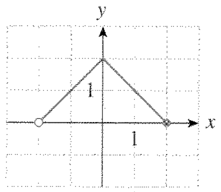
(IV)



(V)



(VI)



In Exercises 7–12, graph the given functions. Give the technology formula and use technology to check your graph. We suggest that you become familiar with these graphs, in addition to those in Table 2. hint [see Example 2]

7. ⊛ $f(x) = -x^3$ (domain $(-\infty, +\infty)$)

8. ⊛ $f(x) = x^3$ (domain $[0, +\infty)$)

9. ⊛ $f(x) = x^4$ (domain $(-\infty, +\infty)$)

10. ⊛ $f(x) = \sqrt[3]{x}$ (domain $(-\infty, +\infty)$)

11. ⊛ $f(x) = \frac{1}{x^2}$ $(x \ne 0)$

12. ⊛ $f(x) = x + \frac{1}{x}$ $(x \ne 0)$

In Exercises 13–18, sketch the graph of the given function, evaluate the given expressions, and then use technology to duplicate the graphs. Give the technology formula. hint [see Example 3]

13. ⊛ $f(x) = \begin{cases} x & \text{if } -4 \le x < 0 \\ 2 & \text{if } 0 \le x \le 4 \end{cases}$

a. $f(-1)$ **b.** $f(0)$ **c.** $f(1)$

14. ⊛ $f(x) = \begin{cases} -1 & \text{if } -4 \le x \le 0 \\ x & \text{if } 0 < x \le 4 \end{cases}$

a. $f(-1)$ **b.** $f(0)$ **c.** $f(1)$

15. ⊛ $f(x) = \begin{cases} x^2 & \text{if } -2 < x \le 0 \\ 1/x & \text{if } 0 < x \le 4 \end{cases}$

a. $f(-1)$ **b.** $f(0)$ **c.** $f(1)$

16. ⊛ $f(x) = \begin{cases} -x^2 & \text{if } -2 < x \le 0 \\ \sqrt{x} & \text{if } 0 < x < 4 \end{cases}$

a. $f(-1)$ **b.** $f(0)$ **c.** $f(1)$

17. ⊛ $f(x) = \begin{cases} x & \text{if } -1 < x \le 0 \\ x + 1 & \text{if } 0 < x \le 2 \\ x & \text{if } 2 < x \le 4 \end{cases}$ hint [see Example 4]

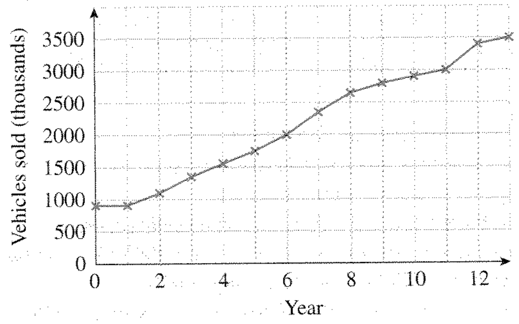
a. $f(0)$ **b.** $f(1)$ **c.** $f(2)$ **d.** $f(3)$

18. ⊛ $f(x) = \begin{cases} -x & \text{if } -1 < x < 0 \\ x - 2 & \text{if } 0 \le x \le 2 \\ -x & \text{if } 2 < x \le 4 \end{cases}$

a. $f(0)$ **b.** $f(1)$ **c.** $f(2)$ **d.** $f(3)$

Applications

Sales of Sport Utility Vehicles *Exercises 19–22 refer to the following graph, which shows the number $f(t)$ of sports utility vehicles (SUVs) sold in the U.S. each year from 1990 through 2003 ($t = 0$ represents 1990, and $f(t)$ represents sales in year t in thousands of vehicles).*[17]



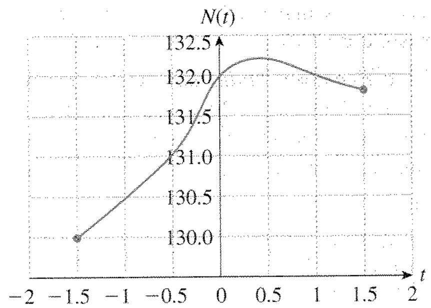
19. ⊛ Estimate $f(6)$, $f(9)$, and $f(7.5)$. Interpret your answers.

20. ⊛ Estimate $f(5)$, $f(11)$, and $f(1.5)$. Interpret your answers.

21. ⊛ Which is larger: $f(6) - f(5)$ or $f(10) - f(9)$? Interpret the answer.

22. ⊛ Which is larger: $f(10) - f(8)$ or $f(13) - f(11)$? Interpret the answer.

23. ⊛ ***Employment*** The following graph shows the number $N(t)$ of people, in millions, employed in the U.S. (t is time in years, and $t = 0$ represents January 2000).[18]



[17] 2000–2003 values were forecasts. Sources: Ford Motor Company/*The New York Times*, February 9, 1995, p. D17, Oak Ridge National Laboratory, Light Vehicle MPG and Market Shares System, AutoPacific, *The U.S. Car and Light Truck Market*, 1999, pp. 24, 120, 121.

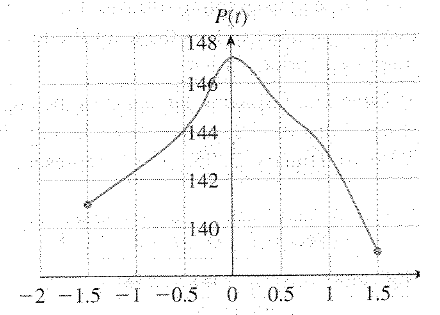
[18] SOURCE: Haver Analytics: The Conference Board/*New York Times*, November 24, 2001.

a. What is the domain of N?

b. Estimate $N(-0.5)$, $N(0)$, and $N(1)$. Interpret answers.

c. On which interval is $N(t)$ falling? Interpret th

24. ⊛ ***Productivity*** The following graph shows an in productivity in the U.S., where t is time in year represents January 2000.[19]



a. What is the domain of P?

b. Estimate $P(-0.5)$, $P(0)$, and $P(1.5)$. Interpre answers.

c. On which interval is $P(t) \ge 144$? Interpret th

25. ⊛ ***Soccer Gear*** The East Coast College soccer ning to buy new gear for its road trip to Califor per shirt depends on the number of shirts the te shown in the following table:

x (Shirts ordered)	5	25	40
A(x) (Cost/shirt, $)	22.91	21.81	21.25 2

a. Which of the following functions best models data?

(A) $A(x) = 0.005x + 20.75$

(B) $A(x) = 0.01x + 20 + \frac{25}{x}$

(C) $A(x) = 0.0005x^2 - 0.07x + 23.25$

(D) $A(x) = 25.5(1.08)^{(x-5)}$

b. tech Ex Graph the model you chose in part $10 \le x \le 100$. Use your graph to estimate th per shirt and the number of shirts the team sh obtain the lowest price per shirt.

26. ⊛ ***Hockey Gear*** The South Coast College hocke to purchase wool hats for its road trip to Alaska hat depends on the number of hats the team ord in the following table:

x (Hats ordered)	5	25	40
A(x) (Cost/hat $)	25.50	23.50	24.63 3

[19] Ibid.

a. Which of the following functions best models the data?
(A) $A(x) = 0.05x + 20.75$
(B) $A(x) = 0.1x + 20 + \frac{25}{x}$
(C) $A(x) = 0.0008x^2 - 0.07x + 23.25$
(D) $A(x) = 25.5(1.08)^{(x-5)}$

b. techEx Graph the model you chose in part (a) with $5 \le x \le 30$. Use your graph to estimate the lowest cost per hat and the number of hats the team should order to obtain the lowest price per hat.

27. ⊛ ***Value of Euro*** The following table shows the approximate value V of one Euro in U.S. dollars from its introduction in January 2000 to January 2005. ($t = 0$ represents January, 2000.)[20]

t (Year)	0	2	5
V (Value in \$)	1.00	0.90	1.30

Which of the following kinds of models would best fit the given data? Explain your choice of model. (A, a, b, c, and m are constants.)

(A) Linear: $V(t) = mt + b$
(B) Quadratic: $V(t) = at^2 + bt + c$
(C) Exponential: $V(t) = Ab^t$

28. ⊛ ***Household Income*** The following table shows the approximate average household income in the U.S. in 1990, 1995, and 2003. ($t = 0$ represents 1990.)[21]

t (Year)	0	5	13
H (Household Income in \$1000)	30	35	43

Which of the following kind of model would best fit the given data? Explain your choice of model. (A, a, b, c, and m are constants.)

(A) Linear: $H(t) = mt + b$
(B) Quadratic: $H(t) = at^2 + bt + c$
(C) Exponential: $H(t) = Ab^t$

29. techEx ***Acquisition of Language*** The percentage $p(t)$ of children who can speak at least single words by the age of t months can be approximated by the equation[22]

$$p(t) = 100\left(1 - \frac{12{,}200}{t^{4.48}}\right) \quad (t \ge 8.5)$$

a. Give a technology formula for p.
b. Graph p for $8.5 \le t \le 20$ and $0 \le p \le 100$. Use your graph to answer parts (c) and (d).
c. What percentage of children can speak at least single words by the age of 12 months? (Round your answer to the nearest percentage point.)
d. By what age are 90% of children speaking at least single words? (Round your answer to the nearest month.)

30. techEx ***Acquisition of Language*** The percentage $p(t)$ of children who can speak in sentences of five or more words by the age of t months can be approximated by the equation[23]

$$p(t) = 100\left(1 - \frac{5.27 \times 10^{17}}{t^{12}}\right) \quad (t \ge 30)$$

a. Give a technology formula for p.
b. Graph p for $30 \le t \le 45$ and $0 \le p \le 100$. Use your graph to answer parts (b) and (c).
c. What percentage of children can speak in sentences of five or more words by the age of 36 months? (Round your answer to the nearest percentage point.)
d. By what age are 75% of children speaking in sentences of five or more words? (Round your answer to the nearest month.)

31. ⊛ ***Processor Speeds*** (Compare Exercise 33 in Section 1.1.) The processor speed, in megahertz, of Intel processors could be approximated by the following function of time t in years since the start of 1995:[24]

$$P(t) = \begin{cases} 75t + 200 & \text{if } 0 \le t \le 4 \\ 600t - 1900 & \text{if } 4 < t \le 9 \end{cases}$$

Sketch the graph of P and use your graph to estimate when processor speeds first reached 2.0 gigahertz (1 gigahertz = 1000 megahertz).

32. ⊛ ***Leading Economic Indicators*** (Compare Exercise 34 in Section 1.1.) The value of the Conference Board Index of 10 economic indicators in the U.S. could be approximated by the following function of time t in months since the end of 2002:[25]

$$E(t) = \begin{cases} 0.4t + 110 & \text{if } 6 \le t \le 15 \\ -0.2t + 119 & \text{if } 15 < t \le 20 \end{cases}$$

Sketch the graph of E and use your graph to estimate when the index first reached 115.

[20] Sources: Bloomberg Financial Markets, International Monetary Fund/*New York Times*, May 18, 2003, p. 17

[21] In current dollars, unadjusted for inflation. Source: U.S. Census Bureau; "Table H-5. Race and Hispanic Origin of Householder—Households by Median and Mean Income: 1967 to 2003;" published August 27, 2004; www.census.gov

[22] The model is the authors' and is based on data presented in the article *The Emergence of Intelligence* by William H. Calvin, *Scientific American*, October, 1994, pp. 101–107.

[23] Ibid.

[24] Source: Sandpile.org/*New York Times*, May 17, 2004, p. C1.

[25] Source: The Conference Board/*New York Times*, November 19, 2004, p. C7.

33. techEx ***Television Advertising*** The cost, in millions of dollars, of a 30-second television ad during the Super Bowl in the years 1990–2001 can be approximated by the following piecewise linear function ($t = 0$ represents 1990):[26]

$$C(t) = \begin{cases} 0.08t + 0.6 & \text{if } 0 \le t < 8 \\ 0.355t - 1.6 & \text{if } 8 \le t \le 11 \end{cases}$$

a. Give a technology formula for C and use technology to graph the function C.
b. Based on the graph, a Superbowl ad first exceeded \$2 million in what year?

34. techEx ***Internet Purchases*** The percentage $p(t)$ of buyers of new cars who used the Internet for research or purchase each year since 1997 is given by the following function[27] ($t = 0$ represents 1997):

$$p(t) = \begin{cases} 10t + 15 & \text{if } 0 \le t < 1 \\ 15t + 10 & \text{if } 1 \le t \le 4 \end{cases}$$

a. Give a technology formula for p and use technology to graph the function p.
b. Based on the graph, 50% or more of all new car buyers used the Internet for research or purchase in what years?

[26] Source: *New York Times*, January 26, 2001, p. C1.

[27] Model is based on data through 2000 (the 2000 value is estimated). Source: J.D. Power Associates/*The New York Times*, January 25, 2000, p. C1.

Communication and Reasoning Exercises

35. ⊛ True or false: Every graphically specified fun
be specified numerically. Explain.

36. ⊛ True or false: Every algebraically specified
also be specified graphically. Explain.

37. ⊛ True or false: Every numerically specified fun
main [0, 10] can also be specified graphically. E

38. ⊛ True or false: Every graphically specified fun
be specified algebraically. Explain.

39. ⊛ How do the graphs of two functions differ if t
fied by the same formula but have different don

40. ⊛ How do the graphs of two functions $f(x)$ anc
$g(x) = f(x) + 10$? (Try an example.)

41. How do the graphs of two functions $f(x)$ and
$g(x) = f(x - 5)$? (Try an example.)

42. How do the graphs of two functions $f(x)$ and
$g(x) = f(-x)$? (Try an example.)

1.3 Linear Functions

Linear functions are among the simplest functions and are perhaps the mos
mathematical functions.

Linear Function

A **linear function** is one that can be written in the form

$f(x) = mx + b$ Function form

or

$y = mx + b$ Equation form

where m and b are fixed numbers (the names m and b are traditional*).

* Actually, c is sometimes used instead of b. As for m, there has even been some research lately into the question of its origin, but no one knows exactly why the letter m is used.

quick Exam

$f(x) = 3x -$

$y = 3x -$