a. Which of the following functions best models the data?

(A) $A(x) = 0.05x + 20.75$

(B) $A(x) = 0.1x + 20 + \dfrac{25}{x}$

(C) $A(x) = 0.0008x^2 - 0.07x + 23.25$

(D) $A(x) = 25.5(1.08)^{(x-5)}$

b. techEx Graph the model you chose in part (a) with $5 \le x \le 30$. Use your graph to estimate the lowest cost per hat and the number of hats the team should order to obtain the lowest price per hat.

27. ⊛ ***Value of Euro*** The following table shows the approximate value V of one Euro in U.S. dollars from its introduction in January 2000 to January 2005. ($t = 0$ represents January, 2000.)[20]

t (Year)	0	2	5
V (Value in \$)	1.00	0.90	1.30

Which of the following kinds of models would best fit the given data? Explain your choice of model. (A, a, b, c, and m are constants.)

(A) Linear: $V(t) = mt + b$

(B) Quadratic: $V(t) = at^2 + bt + c$

(C) Exponential: $V(t) = Ab^t$

28. ⊛ ***Household Income*** The following table shows the approximate average household income in the U.S. in 1990, 1995, and 2003. ($t = 0$ represents 1990.)[21]

t (Year)	0	5	13
H (Household Income in \$1000)	30	35	43

Which of the following kind of model would best fit the given data? Explain your choice of model. (A, a, b, c, and m are constants.)

(A) Linear: $H(t) = mt + b$

(B) Quadratic: $H(t) = at^2 + bt + c$

(C) Exponential: $H(t) = Ab^t$

29. techEx ***Acquisition of Language*** The percentage $p(t)$ of children who can speak at least single words by the age of t months can be approximated by the equation[22]

$$p(t) = 100\left(1 - \frac{12{,}200}{t^{4.48}}\right) \quad (t \ge 8.5)$$

[20] SOURCES: Bloomberg Financial Markets, International Monetary Fund/*New York Times*, May 18, 2003, p. 17

[21] In current dollars, unadjusted for inflation. SOURCE: U.S. Census Bureau; "Table H-5. Race and Hispanic Origin of Householder—Households by Median and Mean Income: 1967 to 2003;" published August 27, 2004; www.census.gov

[22] The model is the authors' and is based on data presented in the article *The Emergence of Intelligence* by William H. Calvin, *Scientific American*, October, 1994, pp. 101–107.

a. Give a technology formula for p.

b. Graph p for $8.5 \le t \le 20$ and $0 \le p \le 100$. Use your graph to answer parts (c) and (d).

c. What percentage of children can speak at least single words by the age of 12 months? (Round your answer to the nearest percentage point.)

d. By what age are 90% of children speaking at least single words? (Round your answer to the nearest month.)

30. techEx ***Acquisition of Language*** The percentage $p(t)$ of children who can speak in sentences of five or more words by the age of t months can be approximated by the equation[23]

$$p(t) = 100\left(1 - \frac{5.27 \times 10^{17}}{t^{12}}\right) \quad (t \ge 30)$$

a. Give a technology formula for p.

b. Graph p for $30 \le t \le 45$ and $0 \le p \le 100$. Use your graph to answer parts (b) and (c).

c. What percentage of children can speak in sentences of five or more words by the age of 36 months? (Round your answer to the nearest percentage point.)

d. By what age are 75% of children speaking in sentences of five or more words? (Round your answer to the nearest month.)

31. ⊛ ***Processor Speeds*** (Compare Exercise 33 in Section 1.1.) The processor speed, in megahertz, of Intel processors could be approximated by the following function of time t in years since the start of 1995:[24]

$$P(t) = \begin{cases} 75t + 200 & \text{if } 0 \le t \le 4 \\ 600t - 1900 & \text{if } 4 < t \le 9 \end{cases}$$

Sketch the graph of P and use your graph to estimate when processor speeds first reached 2.0 gigahertz (1 gigahertz = 1000 megahertz).

32. ⊛ ***Leading Economic Indicators*** (Compare Exercise 34 in Section 1.1.) The value of the Conference Board Index of 10 economic indicators in the U.S. could be approximated by the following function of time t in months since the end of 2002:[25]

$$E(t) = \begin{cases} 0.4t + 110 & \text{if } 6 \le t \le 15 \\ -0.2t + 119 & \text{if } 15 < t \le 20 \end{cases}$$

Sketch the graph of E and use your graph to estimate when the index first reached 115.

[23] Ibid.

[24] SOURCE: Sandpile.org/*New York Times*, May 17, 2004, p. C1.

[25] SOURCE: The Conference Board/*New York Times*, November 19, 2004, p. C7.

33. techEx ***Television Advertising*** The cost, in millions of dollars, of a 30-second television ad during the Super Bowl in the years 1990–2001 can be approximated by the following piecewise linear function ($t = 0$ represents 1990):[26]

$$C(t) = \begin{cases} 0.08t + 0.6 & \text{if } 0 \le t < 8 \\ 0.355t - 1.6 & \text{if } 8 \le t \le 11 \end{cases}$$

a. Give a technology formula for C and use technology to graph the function C.

b. Based on the graph, a Superbowl ad first exceeded \$2 million in what year?

34. techEx ***Internet Purchases*** The percentage $p(t)$ of buyers of new cars who used the Internet for research or purchase each year since 1997 is given by the following function[27] ($t = 0$ represents 1997):

$$p(t) = \begin{cases} 10t + 15 & \text{if } 0 \le t < 1 \\ 15t + 10 & \text{if } 1 \le t \le 4 \end{cases}$$

a. Give a technology formula for p and use technology to graph the function p.

b. Based on the graph, 50% or more of all new car buyers used the Internet for research or purchase in what years?

[26] SOURCE: *New York Times*, January 26, 2001, p. C1.

[27] Model is based on data through 2000 (the 2000 value is estimated). SOURCE: J.D. Power Associates/*The New York Times*, January 25, 2000, p. C1.

Communication and Reasoning Exercises

35. ⊛ True or false: Every graphically specified fun be specified numerically. Explain.

36. ⊛ True or false: Every algebraically specified also be specified graphically. Explain.

37. ⊛ True or false: Every numerically specified fun main [0, 10] can also be specified graphically. E

38. ⊛ True or false: Every graphically specified fun be specified algebraically. Explain.

39. ⊛ How do the graphs of two functions differ if t fied by the same formula but have different don

40. ⊛ How do the graphs of two functions $f(x)$ anc $g(x) = f(x) + 10$? (Try an example.)

41. How do the graphs of two functions $f(x)$ and $g(x) = f(x - 5)$? (Try an example.)

42. How do the graphs of two functions $f(x)$ and $g(x) = f(-x)$? (Try an example.)

1.3 Linear Functions

Linear functions are among the simplest functions and are perhaps the most mathematical functions.

Linear Function

A **linear function** is one that can be written in the form

$f(x) = mx + b$ Function form

or

$y = mx + b$ Equation form

where m and b are fixed numbers (the names m and b are traditional*).

* Actually, c is sometimes used instead of b. As for m, there has even been some research lately into the question of its origin, but no one knows exactly why the letter m is used.

quick Exam

$f(x) = 3x -$

$y = 3x -$

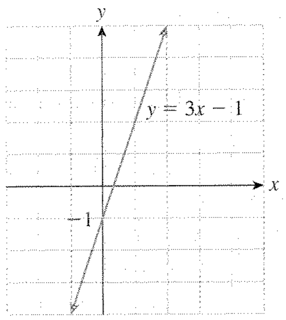


Figure 7

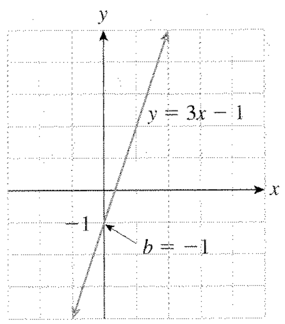


Figure 8
y-intercept = $b = -1$
Graphically, b is the y-intercept of the graph

Linear Functions from the Numerical and Graphical Point of View

The following table shows values of $y = 3x - 1$ ($m = 3$, $b = -1$) for some values of x:

x	−4	−3	−2	−1	0	1	2	3	4
y	−13	−10	−7	−4	−1	2	5	8	11

Its graph is shown in Figure 7.

Looking first at the table, notice that that setting $x = 0$ gives $y = -1$, the value of b.

Numerically, b is the value of y when $x = 0$

On the graph, the corresponding point $(0, -1)$ is the point where the graph crosses the y-axis, and we say that $b = -1$ is the ***y*-intercept** of the graph (Figure 8).

What about m? Looking once again at the table, notice that y increases by $m = 3$ units for every increase of 1 unit in x. This is caused by the term $3x$ in the formula: for every increase of 1 in x we get an increase of $3 \times 1 = 3$ in y.

Numerically, y increases by m units for every 1-unit increase of x

Likewise, for every increase of 2 in x we get an increase of $3 \times 2 = 6$ in y. In general, if x increases by some amount, y will increase by three times that amount. We write:

Change in $y = 3 \times$ Change in x

The Change in a Quantity: Delta Notation

If a quantity q changes from q_1 to q_2, the **change in** q is just the difference:

$$\text{Change in } q = \text{Second value} - \text{First value} = q_2 - q_1$$

Mathematicians traditionally use Δ (delta, the Greek equivalent of the Roman letter D) to stand for change, and write the change in q as Δq.

$$\Delta q = \text{Change in } q = q_2 - q_1$$

quick Examples

1. If x is changed from 1 to 3, we write

$$\Delta x = \text{Second value} - \text{First value} = 3 - 1 = 2$$

2. Looking at our linear function, we see that when x changes from 1 to 3, y changes from 2 to 8. So,

$$\Delta y = \text{Second value} - \text{First value} = 8 - 2 = 6$$

Using delta notation, we can now write, for our linear function,

$$\Delta y = 3\Delta x \qquad \text{Change in } y = 3 \times \text{Change in } x$$

or

$$\frac{\Delta y}{\Delta x} = 3$$

Because the value of y increases by exactly 3 units for every increase of 1 unit in x, the graph is a straight line rising by 3 units for every 1 unit we go to the right. We say that

we have a **rise** of 3 units for each **run** of 1 unit. Because the value c by $\Delta y = 3\Delta x$ units for every change of Δx units in x, in general we h $\Delta y = 3\Delta x$ units for each run of Δx units (Figure 9). Thus, we have a rise c of 2, a rise of 9 for a run of 3, and so on. So, $m = 3$ is a measure of the ste line; we call m the **slope of the line:**

$$\text{Slope} = m = \frac{\Delta y}{\Delta x} = \frac{\text{Rise}}{\text{Run}}$$

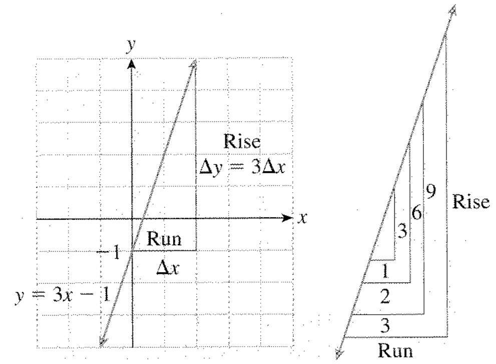


Figure 9
Slope = $m = 3$
Graphically, m is the slope of the graph.

In general (replace the number 3 by a general number m), we can say the f

The Roles of *m* and *b* in the Linear Function $f(x) = mx + b$

Role of *m*

Numerically If $y = mx + b$, then y changes by m units for every 1-unit c change of Δx units in x results in a change of $\Delta y = m\Delta x$ units in y. Thus

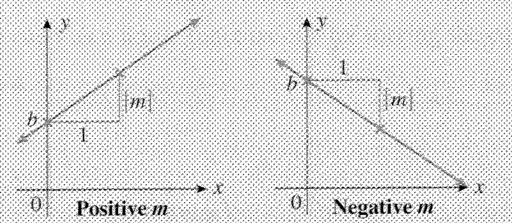
$$m = \frac{\Delta y}{\Delta x} = \frac{\text{Change in } y}{\text{Change in } x}$$

Graphically m is the slope of the line $y = mx + b$:

$$m = \frac{\Delta y}{\Delta x} = \frac{\text{Rise}}{\text{Run}} = \text{Slope}$$

For positive m, the graph rises m units for every 1-unit move to the rig $\Delta y = m\Delta x$ units for every Δx units moved to the right. For negative m, th $|m|$ units for every 1-unit move to the right, and drops $|m|\Delta x$ units for ev moved to the right.

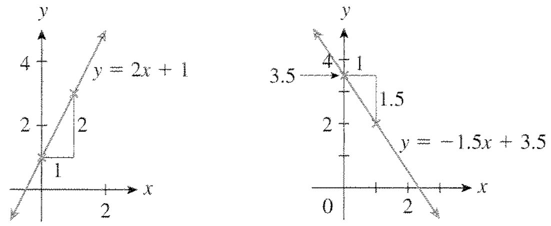
Graph of $y = mx + b$



Role of b
Numerically When $x = 0$, $y = b$
Graphically b is the y-intercept of the line $y = mx + b$.

quick Examples

1. $f(x) = 2x + 1$ has slope $m = 2$ and y-intercept $b = 1$. To sketch the graph, we start at the y-intercept $b = 1$ on the y-axis, and then move 1 unit to the right and up $m = 2$ units to arrive at a second point on the graph. Now connect the two points to obtain the graph on the left.



2. The line $y = -1.5x + 3.5$ has slope $m = -1.5$ and y-intercept $b = 3.5$. Since the slope is negative, the graph (above right) goes *down* 1.5 units for every 1 unit it moves to the right.

It helps to be able to picture what different slopes look like, as in Figure 10. Notice that the larger the absolute value of the slope, the steeper is the line.

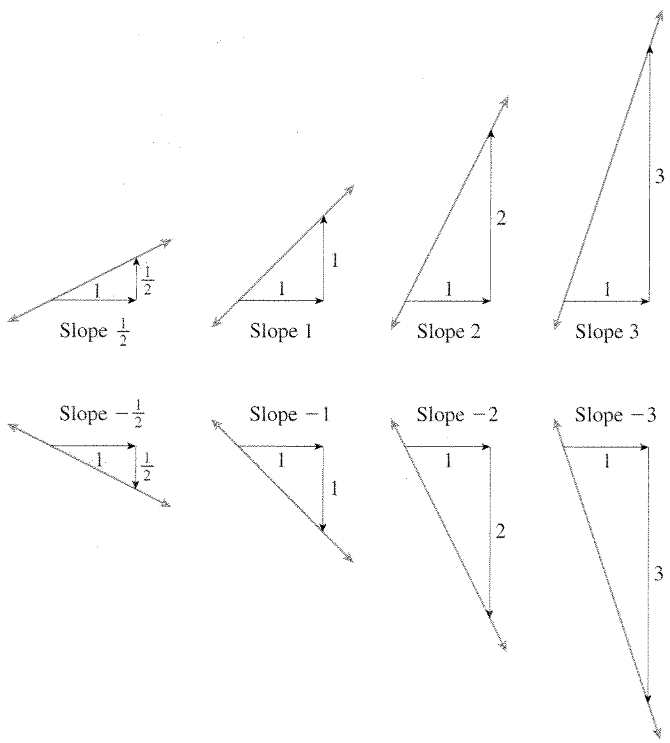


Figure 10

Example 1 Recognizing Linear Data Numerically and Graph

Which of the following two tables gives the values of a linear function? W formula for that function?

x	0	2	4	6	8	10	12
$f(x)$	3	−1	−3	−6	−8	−13	−15

x	0	2	4	6	8	10	12
$g(x)$	3	−1	−5	−9	−13	−17	−21

using *Technology*

Consult the Technology Guides at the end of the chapter to see how to generate tables showing the ratios $\Delta f/\Delta x$ and $\Delta g/\Delta x$. These tables show at a glance that f is not linear.

Solution The function f cannot be linear: If it were, we would have $\Delta f =$ some fixed number m. However, although the change in x between successive the table is $\Delta x = 2$ each time, the change in f is not the same each time. Thu $\Delta f/\Delta x$ is not the same for every successive pair of points.

On the other hand, the ratio $\Delta g/\Delta x$ is the same each time, namely,

$$\frac{\Delta g}{\Delta x} = \frac{-4}{2} = -2$$

Δx		$2-0=2$	$4-2=2$	$6-4=2$	$8-6=2$	$10-8=2$	12 -
x	0	2	4	6	8	10	
$g(x)$	3	−1	−5	−9	−13	−17	
Δg		$(-1)-3 = -4$	$-5-(-1) = -4$	$-9-(-5) = -4$	$-13-(-9) = -4$	$-17-(-13) = -4$	−21

Thus, g is linear with slope $m = -2$. By the table, $g(0) = 3$, hence $b = 3$. T

$$g(x) = -2x + 3$$

Check that this formula gives the values in the ta

If you graph the points in the tables defining f and g above, it becomes easy g is linear and f is not; the points of g lie on a straight line (with slope −2), v points of f do not lie on a straight line (Figure 11).

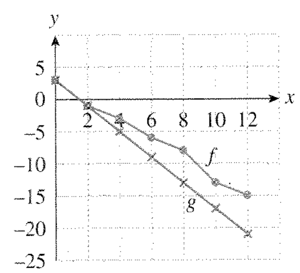


Figure 11

Example 2 Graphing a Linear Equation by Hand: Intercepts

Graph the equation $x + 2y = 4$. Where does the line cross the x- and y-axes

Solution We first write y as a linear function of x by solving the equation f

$$2y = -x + 4$$

so

$$y = -\frac{1}{2}x + 2$$

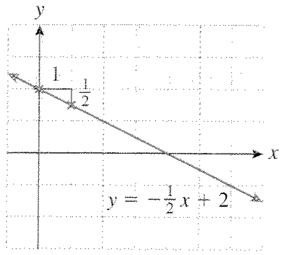


Figure 12

Now we can see that the graph is a straight line with a slope of $-1/2$ and a y-intercept of 2. We start at 2 on the y-axis and go down 1/2 unit for every 1 unit we go to the right. The graph is shown in Figure 12.

We already know that the line crosses the y-axis at 2. Where does it cross the x-axis? Wherever that is, we know that the y-coordinate will be 0 at that point. So, we set $y = 0$ and solve for x. It's most convenient to use the equation we were originally given:

$$x + 2(0) = 4 \qquad \text{Original equation with } x = 0$$

so $$x = 4$$

The line crosses the x-axis at 4.

Before we go on... We could have graphed the equation in Example 2 another way, by first finding the intercepts. Once we know that the line crosses the y-axis at 2 and the x-axis at 4, we can draw those two points and then draw the line connecting them.

We now summarize the procedure for finding the intercepts of a line.

Finding the Intercepts

The **x-intercept** of a line is where it crosses the x-axis. To find it, set $y = 0$ and solve for x. The **y-intercept** is where it crosses the y-axis. If the equation of the line is written as $y = mx + b$, then b is the y-intercept. Otherwise, set $x = 0$ and solve for y.

quick Example

Consider the equation $3x - 2y = 6$. To find its x-intercept, set $y = 0$ to find $x = 6/3 = 2$. To find its y-intercept, set $x = 0$ to find $y = 6/(-2) = -3$. The line crosses the x-axis at 2 and the y-axis at -3.

Computing the Slope of a Line

We know that the slope of a line is given by

$$\text{Slope} = m = \frac{\text{Rise}}{\text{Run}} = \frac{\Delta y}{\Delta x}$$

Recall that two points—say (x_1, y_1) and (x_2, y_2)—determine a line in the xy-plane. To find its slope, we need a run Δx and corresponding rise Δy. In Figure 13, we see that we can use $\Delta x = x_2 - x_1$, the change in the x-coordinate from the first point to the second, as our run, and $\Delta y = y_2 - y_1$, the change in the y-coordinate, as our rise. The resulting formula for computing the slope is given below.

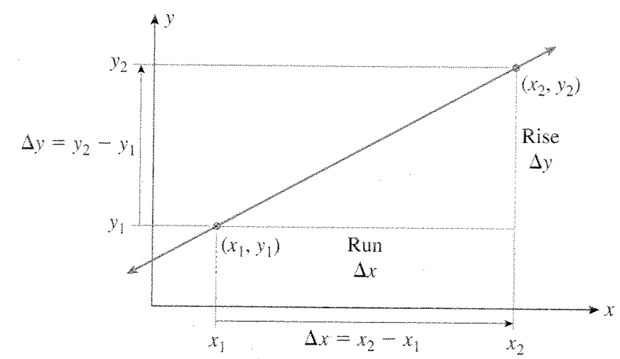


Figure 13

Computing the Slope of a Line

We can compute the slope m of the line through the points (x_1, y_1) and (

$$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$$

quick Examples

1. The slope of the line through $(x_1, y_1) = (1, 3)$ and $(x_2, y_2) = (5, 11)$ is

$$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{11 - 3}{5 - 1} = \frac{8}{4} = 2$$

2. The slope of the line through $(x_1, y_1) = (1, 2)$ and $(x_2, y_2) = (2, 1)$ is

$$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{1 - 2}{2 - 1} = \frac{-1}{1} = -1$$

Q: *What if we had chosen to list the two points in Quick Example 1 in reverse orde suppose we had taken $(x_1, y_1) = (5, 11)$ and $(x_2, y_2) = (1, 3)$. What would have bee on the computation of the slope?*

A: We would have found

$$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{3 - 11}{1 - 5} = \frac{-8}{-4} = 2$$

the same answer. The order in which we take the points is not important, *as long as same order in the numerator and the denominator.*

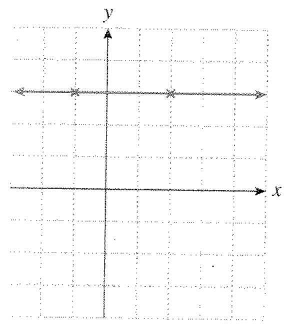
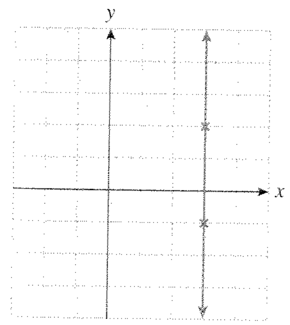


Figure 14



Vertical lines have undefined slope.

Figure 15

Example 3 Special Slopes

Find the slope of the line through (2, 3) and (−1, 3) and the slope of the (3, 2) and (3, −1).

Solution The line through (2, 3) and (−1, 3) has slope

$$m = \frac{\Delta y}{\Delta x} = \frac{3 - 3}{-1 - 2} = \frac{0}{-3} = 0$$

A line of slope 0 has 0 rise, so is a *horizontal* line, as shown in Figure 1 through (3, 2) and (3, −1) has slope

$$m = \frac{\Delta y}{\Delta x} = \frac{-1 - 2}{3 - 3} = \frac{-3}{0}$$

which is undefined. If we plot the two points, we see that the line passing th is *vertical*, as shown in Figure 15.

Finding a Linear Equation from Data: How to Make a Linear Model

If we happen to know the slope and y-intercept of a line, writing down its straightforward. For example, if we know that the slope is 3 and the y-inter then the equation is $y = 3x - 1$. Sadly, the information we are given is seldom so convenient. For instance, we may know the slope and a point other than the y-intercept, two points on the line, or other information.

We describe a straightforward method for finding the equation of a line: the **point-slope** method. As the name suggests, we need two pieces of information:

- The *slope* m (which specifies the direction of the line)
- A *point* (x_1, y_1) on the line (which pins down its location in the plane)

The equation of the line through the point (x_1, y_1) with slope m must have the form

$$y = mx + b$$

for some (unknown) number b. To determine b we use the fact that the line must pass through the point (x_1, y_1), so that (x_1, y_1) satisfies the equation $y = mx + b$. In other words,

$$y_1 = mx_1 + b$$

Solving for b gives

$$b = y_1 - mx_1$$

Summarizing:

The Point-Slope Formula

An equation of the line through the point (x_1, y_1) with slope m is given by

$$y = mx + b \quad \text{Equation form}$$

where

$$b = y_1 - mx_1$$

quick Example The line through (2, 3) with slope 4 has equation

$$y = 4x + b, \quad \text{where } b = 3 - (4)(2) = -5, \quad \text{so } y = 4x - 5$$

Q: *When do we use the point-slope formula rather than the slope-intercept form?*

A: Use the point-slope formula to find the equation of a line when you are given information about a point and the slope of the line. The formula does not apply if the slope is undefined, as in a vertical line; see Example 4(d) below. The slope-intercept form is more useful for graphing a line whose equation you already have. ■

Example 4 Using the Point-Slope Formula

Find equations for the following straight lines.

a. Through the points (1, 2) and (3, −1)

b. Through (2, −2) and parallel to the line $3x + 4y = 5$

c. Horizontal and through (−9, 5)

d. Vertical and through (−9, 5)

Solution In each case other than (d), we apply the point-slope formula.

a. To apply the point-slope formula, we need

- ***Point*** We have two to choose from, so we take the first, $(x_1, y_1) = (1,$
- ***Slope*** Not given directly, but we do have enough information to calcul we are given two points on the line, we can use the slope formula:

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-1-2}{3-1} = -\frac{3}{2}$$

An equation of the line is therefore

$$y = -\frac{3}{2}x + b$$

where $b = y_1 - mx_1 = 2 - \left(-\frac{3}{2}\right)(1) = \frac{7}{2}$, so

$$y = -\frac{3}{2}x + \frac{7}{2}$$

b. Proceeding as before,

- ***Point*** Given here as (2, −2).
- ***Slope*** We use the fact that *parallel lines have the same slope*. (Wh find the slope of $3x + 4y = 5$ by solving for y and then looking at the of x:

$$y = -\frac{3}{4}x + \frac{5}{4} \qquad \text{To find the slope, solve for } y.$$

so the slope is $-3/4$.

An equation for the desired line is

$$y = -\frac{3}{4}x + b$$

where $b = y_1 - mx_1 = -2 - \left(-\frac{3}{4}\right)(2) = -\frac{1}{2}$

so $$y = -\frac{3}{4}x - \frac{1}{2}$$

c. We are given a point: (−9, 5). Furthermore, we are told that the line is which tells us that the slope is 0. Therefore, we get

$$y = 0x + b = b$$

where $b = y_1 - mx_1 = 5 - (0)(-9) = 5$

so $y = 5$

d. We are given a point: (−9, 5). This time, we are told that the line is ver means that the slope is undefined. Thus, we can't use the point-slope for formula makes sense only when the slope of the line is defined.) What Well, here are some points on the desired line:

$$(-9, 1), (-9, 2), (-9, 3), \ldots,$$

so $x = -9$ and $y = $ *anything*. If we simply say that $x = -9$, then these p solutions, so the equation is $x = -9$.

1.3 EXERCISES

⊛ denotes basic skills exercises

tech Ex indicates exercises that should be solved using technology

In Exercises 1–6, a table of values for a linear function is given. Fill in the missing value and calculate m in each case.

1. ⊛

x	-1	0	1
y	5	8	

2. ⊛

x	-1	0	1
y	-1	-3	

3. ⊛

x	2	3	5
$f(x)$	-1	-2	

4. ⊛

x	2	4	5
$f(x)$	-1	-2	

5. ⊛

x	-2	0	2
$f(x)$	4		10

6. ⊛

x	0	3	6
$f(x)$	-1		-5

In Exercises 7–10, first find $f(0)$, if not supplied, and then find the equation of the given linear function.

7. ⊛

x	-2	0	2	4
$f(x)$	-1	-2	-3	-4

8. ⊛

x	-6	-3	0	3
$f(x)$	1	2	3	4

9. ⊛

x	-4	-3	-2	-1
$f(x)$	-1	-2	-3	-4

10. ⊛

x	1	2	3	4
$f(x)$	4	6	8	10

In each of Exercises 11–14, decide which of the two given functions is linear and find its equation. hint [see Example 1]

11. ⊛

x	0	1	2	3	4
$f(x)$	6	10	14	18	22
$g(x)$	8	10	12	16	22

12. ⊛

x	-10	0	10	20	30
$f(x)$	-1.5	0	1.5	2.5	3.5
$g(x)$	-9	-4	1	6	11

13. ⊛

x	0	3	6	10	15
$f(x)$	0	3	5	7	9
$g(x)$	-1	5	11	19	29

14. ⊛

x	0	3	5	6	9
$f(x)$	2	6	9	12	15
$g(x)$	-1	8	14	17	26

In Exercises 15–24, find the slope of the given line, if it is defined.

15. ⊛ $y = -\frac{3}{2}x - 4$

16. ⊛ $y = \frac{2x}{3} + 4$

17. ⊛ $y = \frac{x+1}{6}$

18. ⊛ $y = -\frac{2x-1}{3}$

19. ⊛ $3x + 1 = 0$

20. ⊛ $8x - 2y = 1$

21. ⊛ $3y + 1 = 0$

22. ⊛ $2x + 3 = 0$

23. ⊛ $4x + 3y = 7$

24. ⊛ $2y + 3 = 0$

In Exercises 25–38, graph the given equation. hint [see Example 2]

25. ⊛ $y = 2x - 1$

26. ⊛ $y = x - 3$

27. ⊛ $y = -\frac{2}{3}x + 2$

28. ⊛ $y = -\frac{1}{2}x + 3$

29. ⊛ $y + \frac{1}{4}x = -4$

30. ⊛ $y - \frac{1}{4}x = -2$

31. ⊛ $7x - 2y = 7$

32. ⊛ $2x - 3y = 1$

33. ⊛ $3x = 8$

34. ⊛ $2x = -7$

35. ⊛ $6y = 9$

36. ⊛ $3y = 4$

37. ⊛ $2x = 3y$

38. ⊛ $3x = -2y$

In Exercises 39–54, calculate the slope, if defined, of the straight line through the given pair of points. Try to do as many as you can without writing anything down except the answer. hint [see Example 3]

39. ⊛ $(0, 0)$ and $(1, 2)$

40. ⊛ $(0, 0)$ and $(-1, 2)$

41. ⊛ $(-1, -2)$ and $(0, 0)$

42. ⊛ $(2, 1)$ and $(0, 0)$

43. ⊛ $(4, 3)$ and $(5, 1)$

44. ⊛ $(4, 3)$ and $(4, 1)$

45. ⊛ $(1, -1)$ and $(1, -2)$

46. ⊛ $(-2, 2)$ and $(-1, -1)$

47. ⊛ $(2, 3.5)$ and $(4, 6.5)$

48. ⊛ $(10, -3.5)$ and $(0, -1.5)$

49. ⊛ $(300, 20.2)$ and $(400, 11.2)$

50. ⊛ $(1, -20.2)$ and $(2, 3.2)$

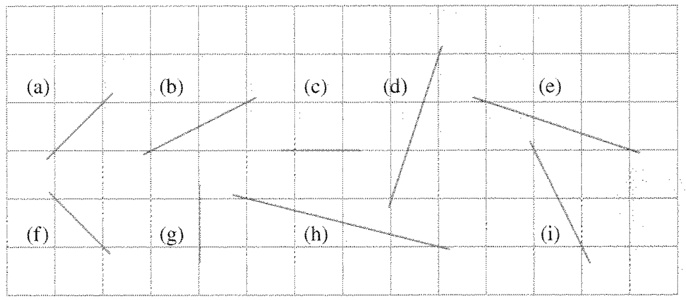
51. ⊛ $(0, 1)$ and $\left(-\frac{1}{2}, \frac{3}{4}\right)$

52. ⊛ $\left(\frac{1}{2}, 1\right)$ and $\left(-\frac{1}{2}, \frac{3}{4}\right)$

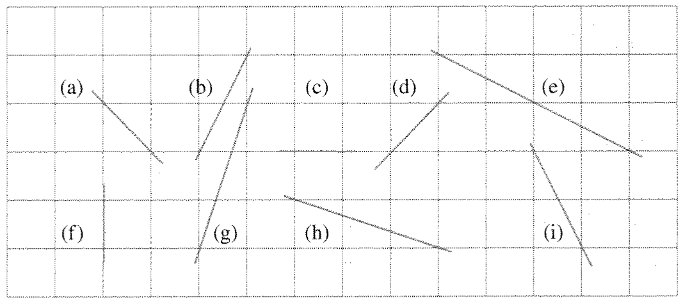
53. ⊛ (a, b) and (c, d) $(a \neq c)$

54. ⊛ (a, b) and (c, b) $(a \neq c)$

55. ⊛ In the following figure, estimate the slopes of all line segments.



56. ⊛ In the following figure, estimate the slopes of all line segments.



In Exercises 57–70, find a linear equation whose graph is the straight line with the given properties. hint [see Example 4]

57. ⊛ Through $(1, 3)$ with slope 3

58. ⊛ Through $(2, 1)$ with slope 2

59. ⊛ Through $\left(1, -\frac{3}{4}\right)$ with slope $\frac{1}{4}$

60. ⊛ Through $\left(0, -\frac{1}{3}\right)$ with slope $\frac{1}{3}$

61. ⊛ Through $(20, -3.5)$ and increasing at a rate of 10 units of y per unit of x

62. ⊛ Through $(3.5, -10)$ and increasing at a rate of 1 unit of y per 2 units of x.

63. ⊛ Through $(2, -4)$ and $(1, 1)$

64. ⊛ Through $(1, -4)$ and $(-1, -1)$

65. ⊛ Through $(1, -0.75)$ and $(0.5, 0.75)$

66. ⊛ Through $(0.5, -0.75)$ and $(1, -3.75)$

67. ⊛ Through $(6, 6)$ and parallel to the line $x + y = 4$

68. ⊛ Through $(1/3, -1)$ and parallel to the line $3x - 4y = 8$

69. ⊛ Through $(0.5, 5)$ and parallel to the line $4x - 2y = 11$

70. ⊛ Through $(1/3, 0)$ and parallel to the line $6x - 2y = 11$

Communication and Reasoning Exercises

71. ⊛ How would you test a table of values of x … comes from a linear function?

72. ⊛ You have ascertained that a table of values … sponds to a linear function. How do you find … that linear function?

73. ⊛ To what linear function of x does the … $ax + by = c\,(b \neq 0)$ correspond? Why d… $b \neq 0$?

74. ⊛ Complete the following. The slope of the li… $y = mx + b$ is the number of units that ___ … unit increase in ____.

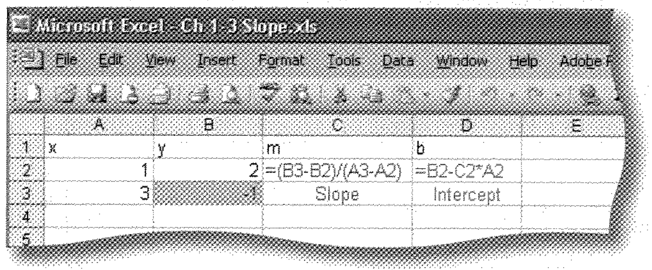
75. ⊛ Complete the following. If, in a straight line … three times as fast as x, then its ____ is ____.

76. ⊛ Suppose that y is decreasing at a rate of 4 un… crease of x. What can we say about the slope … lationship between x and y? What can we … intercept?

77. ⊛ If y and x are related by the linear expression $y = mx + b$, how will y change as x changes if m is positive? negative? zero?

78. ⊛ Your friend April tells you that $y = f(x)$ has the property that, whenever x is changed by Δx, the corresponding change in y is $\Delta y = -\Delta x$. What can you tell her about f?

79. tech Ex Consider the following worksheet:



	A	B	C	D	E
1	x	y	m	b	
2	1	2	=(B3-B2)/(A3-A2)	=B2-C2*A2	
3	3	-1	Slope	Intercept	
4					
5					

What is the effect on the slope of increasing the y-coordinate of the second point (the point whose coordinates are in Row 3)? Explain.

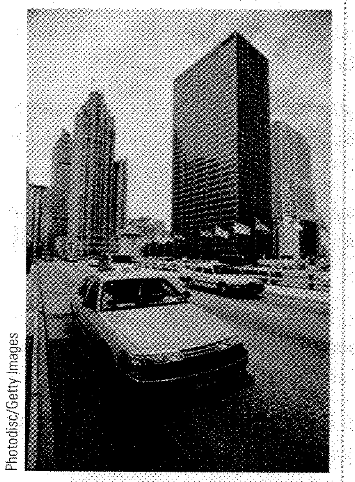
80. tech Ex Referring to the worksheet in Exercise 79, what is the effect on the slope of increasing the x-coordinate of the second point (the point whose coordinates are in row 3)? Explain.

⊛ basic skills   tech Ex technology exercise

1.4 Linear Models

Using linear functions to describe or approximate relationships in the real world is called **linear modeling.** We start with some examples involving cost, revenue, and profit.

Cost, Revenue, and Profit Functions



Photodisc/Getty Images

Example 1 Linear Cost Function

As of January, 2005, Yellow Cab Chicago's rates were \$1.90 on entering the cab plus \$1.60 for each mile.*

a. Find the cost C of an x-mile trip.

b. Use your answer to calculate the cost of a 40-mile trip.

c. What is the cost of the second mile? What is the cost of the tenth mile?

d. Graph C as a function of x.

Solution

a. We are being asked to find how the cost C depends on the length x of the trip, or to find C as a function of x. Here is the cost in a few cases:

Cost of a 1-mile trip: $C = 1.60(1) + 1.90 = 3.50$ 1 mile @ \$1.60 per mile plus \$1.90

Cost of a 2-mile trip: $C = 1.60(2) + 1.90 = 5.10$ 2 miles @ \$1.60 per mile plus \$1.90

Cost of a 3-mile trip: $C = 1.60(3) + 1.90 = 6.70$ 3 miles @ \$1.60 per mile plus \$1.90

Do you see the pattern? The cost of an x-mile trip is given by the linear function:

$$C(x) = 1.60x + 1.90$$

Notice that the slope 1.60 is the incremental cost per mile. In this context we call 1.60 the **marginal cost;** the varying quantity $1.60x$ is called the **variable cost.** The

*According to their website at www.yellowcabchicago.com/.

C-intercept 1.90 is the cost to enter the cab, which we call the **fixed cost.** a linear cost function has the following form:

$$C(x) = \overbrace{mx}^{\text{Variable cost}} + b$$

↑ Marginal cost   ↑ Fixed cost

b. We can use the formula for the cost function to calculate the cost o trip as:

$$C(40) = 1.60(40) + 1.90 = \$65.90$$

c. To calculate the cost of the second mile, we *could* proceed as follows:

Find the cost of a 1-mile trip: $C(1) = 1.60(1) + 1.90 = \$3.50$

Find the cost of a 2-mile trip: $C(2) = 1.60(2) + 1.90 = \$5.10$

Therefore, the cost of the second mile is $\$5.10 - \$3.50 = \$1.60$

But notice that this is just the marginal cost. In fact, the marginal cost is each additional mile, so we could have done this more simply:

Cost of second mile = Cost of tenth mile = Marginal cost = \$1.6

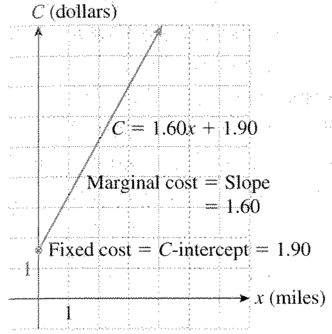


Figure 16

d. Figure 16 shows the graph of the cost function, which we can interpret a *miles* graph. The fixed cost is the starting height on the left, while the mar the slope of the line.

Before we go on... In general, the slope m measures the number of units o y per 1-unit change in x, so *we measure m in units of y per unit of x:*

Units of Slope = Units of y per unit of x

In Example 1, y is the cost C, measured in dollars, and x is the length of a tri in miles. Hence,

Units of Slope = Units of y per Unit of x = Dollars per mile

The y-intercept b, being a value of y, is measured in the same units as y. In b is measured in dollars. ⊛

Here is a summary of the terms used in the preceding example, alo introduction to some new terms.

Cost, Revenue, and Profit Functions

A **cost function** specifies the cost C as a function of the number of ite $C(x)$ is the cost of x items. A cost function of the form

$$C(x) = mx + b$$

is called a **linear cost function.** The quantity mx is called the **variable c** intercept b is called the **fixed cost.** The slope m, the **marginal cost,** measure mental cost per item.