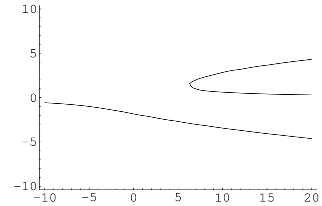
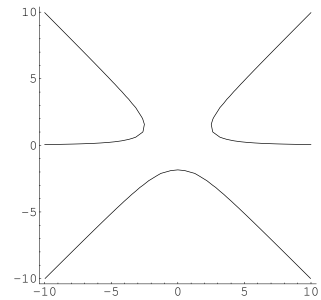
Math 251

Review Problems Name:________________________

Show all relevant work!

1. The graph of the implicit relation $y^3 - xy = -6$ is shown to the right. Show that there is one vertical tangent ($x = 3^{5/3}$) and no horizontal tangent (*careful*).





2. The graph of the implicit relation $y^3 - x^2y = -6$ is shown to the right. Show that there are two vertical tangents ($x = 3^{5/6}$ and $x = -3^{5/6}$) and one horizontal tangent ($y = -6^{1/3}$)).

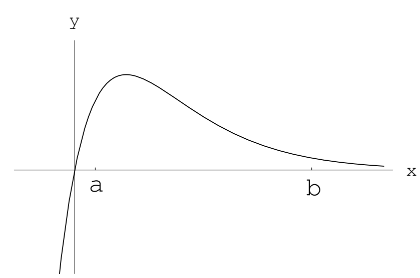
3. Consider the curve $\ln(xy) = x - y$.

(a) Show that $y' = \frac{y(x-1)}{x(y+1)}$

(b) Find the equation of the line tangent to this curve at (1, 1).

4. Write the equation of the horizontal tangent to the curve $2y^3 + 6x^2y - 12x^2 + 6y = 1$.

5. The graph of $f(x) = xe^{-2x}$ is shown below. Note that near point $(a, f(a))$, the tangent approximation overestimates $f(x)$, while near point $(b, f(b))$, the tangent approximation underestimates $f(x)$. Determine the point where the tangent line approximations stop overestimating the value of $f(x)$ and start underestimating $f(x)$.



6. The daily cost, C, of running an air conditioner in Arizona depends on the temperature, H, as shown in the first table. The temperature in turn increases with the time of day, t, as shown in the second table. Determine the approximate rate at which cost changes with time when $t = 10$ and interpret the result.

H (in F°)	90	95	100	105	110
$C(H)$ ($)	4	4.75	6	7.50	9.15

t (in hours past 00:00)	6	8	10	12	14
$H(t)$ (in F°)	90	97	100	112	119

7. Suppose f and g are differentiable functions with values given in the table below. If $h(x) = f(g(x))$, use the table to determine $h'(2)$.

x	2	3	4	5
$f(x)$	5	7	2	2
$g(x)$	5	8	4	8
$f'(x)$	e	4	5	π
$g'(x)$	$\sqrt{2}$	7	9	7

8. Find the derivative of each function.

(a) $f(x) = e^{2\ln\sqrt{x^3-1}}$

(b) $y = \dfrac{x-1}{x^2-x}$

(c) $y = 2^{\pi}$

(d) $y = \dfrac{1}{x\ln x}$

(e) $g(x) = \ln\sqrt{x^2+1}$

(f) $h(x) = x^{1-2x}$

9. Find the derivative of each function.

(a) $f(x) = x\ln x - x$

(b) $y = xe^{\ln x^2}$

(c) $g(x) = 2^{5-x^3}$

(d) $y = \dfrac{\sqrt{x}}{\ln\sqrt{x}}$

(e) $\dfrac{e^{-x^2}}{x}$

(f) $g(x) = \dfrac{e^x - e^{-x}}{e^x + e^{-x}}$

10. Find the equation of the line tangent to $f(x) = \ln\sqrt{2x^2+1}$ at $x = 1$.

11. The population of South Park in 1980 was 50,000. The population in 1995 fell to 38,000.

(a) If you assume the population varies exponentially with time, find a formula for the population as a function of time.

(b) Determine how quickly the population is changing in 1995 (and in what direction).

12. Suppose $f(x)$ is a differentiable function, $f'(x) > 0$ and $f''(x) < 0$ for all $x \in \mathbb{R}$. If $f(3) = 2$ and $f'(3) = 4$, then which of the values below is feasible? Explain.

a) $f(5) = 1$ b) $f(5) = 7$ c) $f(5) = 12$

13. Determine the intervals where $y = e^{-x^2}$ is concave up.

14. Determine the intervals where $y = xe^{-x^2}$ is concave up.

15. Show that $\dfrac{\mathrm{d}}{\mathrm{d}x}\arctan x = \dfrac{1}{1+x^2}$