Review Problems | Solutions

1. $y' = \frac{y}{3y^2-x}$. Note that the only candidate for a horizontal tangent is at $y = 0$ but this leads to $0 = -6$ in the original curve so there is no horizontal tangent. Substituting $x = 3y^2$ into the original curve gives $y = \sqrt[3]{3}$ and this leads to the solution $x = 3^{5/3}$.

2. $y' = \frac{2xy}{3y^2-x^2}$. The horizontal tangent occurs where the numerator of y' is 0 so $2xy = 0$. Then either $x = 0$, $y = 0$ or both. In the case where $y = 0$ (or both) we get $0 = -6$ in the original equation so this is not a solution.
If $x = 0$ we have $y^3 = -6$ so $y = (-6)^{1/3} = -6^{1/3}$ is the equation of the horizontal tangent.
Vertical tangents occur where $3y^2 - x^2 = 0$ which gives us $3y^2 = x^2$ and substituting into the original equation we have $y^3 - 3y^3 = -6 \longrightarrow y = 3^{1/3}$. Then $x^2 = 3(3^{1/3})^2$ so $x = \pm 3^{5/6}$ gives the two vertical tangents.

3. Equation is $y = 1$

4. $y' = \frac{4x-2xy}{y^2+x^2+1}$. The two solutions to $4x - 2xy = 0$ give one superfluous result ($y = 2$ fails to be a point on the curve), and one good one: $x = 0 \longrightarrow y = 0.165$.

5. Hint: What is this asking? Try sketching the tangent lines. The answer is $(1, e^{-2})$.

6. About \$1.03 per hour at 10:00 am.

7. $\pi\sqrt{2}$

8. (a) $f'(x) = 3x^2$ (b) $y' = -x^{-2}$ (c) $y' = 0$ (d) $y' = -\frac{\ln x+1}{(x\ln x)^2}$

(e) $g'(x) = \dfrac{x}{x^2+1}$ (f) $h'(x) = x^{1-2x}\left(\dfrac{-2x\ln x + 1 - 2x}{x}\right) = x^{-2x}(-2x\ln x + 1 - 2x)$

9. (a) $f'(x) = \ln x$ (b) $y' = 3x^2$ (c) $g'(x) = -3\ln 2 \cdot x^2 \cdot 2^{5-x^3}$

(d) $y' = \dfrac{\sqrt{x}\,(\ln\sqrt{x} - 1)}{2x[\ln\sqrt{x}]^2} = \dfrac{\ln x - 2}{\sqrt{x}(\ln x)^2}$ (e) $e^{-x^2}(2 + x^{-2})$ (f) $g(x) = \dfrac{4}{(e^x + e^{-x})^2}$

10. $y = \frac{2}{3}x + (\ln\sqrt{3} - \frac{2}{3})$

11. (a) $P = 50000e^{-0.018296t}$ (b) $\frac{dP}{dt} = -695.2$ so population decreasing by 695 people per year in 1995.

12. Tangent line at $x = 3$ would give an approximation of $f(5) = 10$. Since f is concave down, we know the tangent line is an overestimate so (c) is impossible. Since f is increasing and $f(3) = 2$, (a) is impossible. Therefore (b) is the only reasonable solution.

13. $\left(-\infty, -\sqrt{\frac{1}{2}}\right]$ and $\left[\sqrt{\frac{1}{2}}, \infty\right)$

14. $\left[-\sqrt{\frac{3}{2}}, 0\right]$ and $\left[\sqrt{\frac{3}{2}}, \infty\right)$

15. As with other inverse functions we begin by writing
$\cos(\arccos x) = x$ and differentiating both sides gives
$\frac{d}{dx}\tan(\arctan x) = \frac{d}{dx}x \longrightarrow$
$\sec^2(\arctan x)\cdot\frac{d}{dx}\arctan x = 1$
$\frac{d}{dx}\arctan x = \frac{1}{\sec^2(\arctan x)} = \cos^2(\arctan x)$
and from the diagram this yields
$\frac{d}{dx}\arctan x = \frac{1}{1+x^2}$.