1. $y' = \frac{y}{3y^2-x}$. Note that the only candidate for a horizontal tangent is at $y = 0$ but this leads to $0 = -6$ in the original curve so there is no horizontal tangent. Substituting $x = 3y^2$ into the original curve gives $y = \sqrt[3]{3}$ and this leads to the solution $x = 3^{5/3}$.

2. $y' = \frac{2xy}{y^2-x^2}$. The horizontal tangent occurs where the numerator of $y'$ is 0 so $2xy = 0$. Then either $x = 0$, $y = 0$ or both. In the case where $y = 0$ (or both) we get $0 = -6$ in the original equation so this is not a solution. If $x = 0$ we have $y^3 = -6$ so $y = (-6)^{1/3} = -6^{1/3}$ is the equation of the horizontal tangent. Vertical tangents occur where $3y^2 - x^2 = 0$ which gives us $3y^2 = x^2$ and substituting into the original equation we have $y^3 - 3y^3 = -6 \rightarrow y = 3^{1/3}$. Then $x^2 = 3(3^{1/3})^2$ so $x = \pm 3^{5/6}$ gives the two vertical tangents.

3. Equation is $y = 1$

4. $y' = \frac{4x - 2xy}{x^2 + 2x}$. The two solutions to $4x - 2xy = 0$ give one superfluous result ($y = 2$ fails to be a point on the curve), and one good one: $x = 0 \rightarrow y = 0.165$.

5. Hint: What is this asking? Try sketching the tangent lines. The answer is $(1, e^{-2})$.

6. About $1.03$ per hour at 10:00 am.

7. $\pi \sqrt{2}$

8. (a) $f'(x) = 3x^2$  
(b) $y' = -x^{-2}$  
(c) $y' = 0$  
(d) $y' = -\frac{\ln x + 1}{(x \ln x)^2}$  
(e) $g'(x) = \frac{x}{x^2 + 1}$  
(f) $h'(x) = x^{1-2x} \left(\frac{-2x \ln x + 1 - 2x}{x}\right) = x^{-2x}(-2x \ln x + 1 - 2x)$

9. (a) $f'(x) = \ln x$  
(b) $y' = 3x^2$  
(c) $g'(x) = -3 \ln 2 \cdot x^2 \cdot 2^5 - x^3$  
(d) $y' = \frac{\sqrt{x} (\ln \sqrt{x} - 1)}{2x[\ln \sqrt{x}]^2} = \frac{\ln x - 2}{\sqrt{x}(\ln x)^2}$  
(e) $e^{-x^2}(2 + x^{-2})$  
(f) $g(x) = \frac{4}{(e^x + e^{-x})^2}$

10. $y = \frac{2}{3}x + (\ln \sqrt{3} - \frac{2}{3})$

11. (a) $P = 500000e^{-0.018296t}$  
(b) $\frac{dP}{dt} = -695.2$ so population decreasing by 695 people per year in 1995.

12. Tangent line at $x = 3$ would give an approximation of $f(5) = 10$. Since $f$ is concave down, we know the tangent line is an overestimate so (c) is impossible. Since $f$ is increasing and $f(3) = 2$, (a) is impossible. Therefore (b) is the only reasonable solution.

13. $(-\infty, -\sqrt{\frac{1}{2}}]$ and $[\sqrt{\frac{1}{2}}, \infty)$

14. $[-\sqrt{\frac{1}{2}}, 0]$ and $[\sqrt{\frac{1}{2}}, \infty)$

15. As with other inverse functions we begin by writing

$$
\cos (\arccos x) = x \quad \text{and differentiating both sides gives}
$$

$$
\frac{d}{dx} \tan (\arctan x) = \frac{d}{dx} x \quad \rightarrow
$$

$$
\sec^2 (\arctan x) \cdot \frac{d}{dx} \arctan x = 1
$$

$$
\frac{d}{dx} \arctan x = \frac{1}{1+x^2} \quad \arctan x = \cos^2 (\arctan x)
$$

and from the diagram this yields

$$
\frac{d}{dx} \arctan x = \frac{1}{1+x^2}.
$$