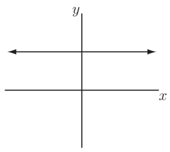
Math 251

3.1 Notes

1. Some elementary derivatives.

$\frac{\mathrm{d}}{\mathrm{d}x}k$: **The derivative of a constant.** Some arguments for $\frac{\mathrm{d}}{\mathrm{d}x}k = 0$:

Graphically:
The derivative gives the slope of the fuction at x.
For a constant function, the slope is 0 so $\frac{\mathrm{d}}{\mathrm{d}x}k = 0$.



Numerically:

x	-3	0	3	6	9
$f(x)$	k	k	k	k	k

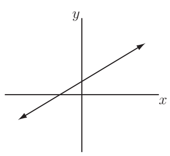
0 0 0 0

Algebraically (from definition):

$$\frac{\mathrm{d}}{\mathrm{d}x}k = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h} = \lim_{h\to 0}\frac{k-k}{h} = \lim_{h\to 0}\frac{0}{h} = 0$$

$\frac{\mathrm{d}}{\mathrm{d}x}(mx+b)$: **The derivative of a linear function.** Some arguments for $\frac{\mathrm{d}}{\mathrm{d}x}(mx+b) = m$:

Graphically:
The derivative gives the slope of the fuction at x.
For a linear function, the slope is m so $\frac{\mathrm{d}}{\mathrm{d}x}(mx+b) = m$.



Numerically:

x	-1	0	1	2	3
$f(x)$	$-m+b$	$0+b$	$m+b$	$2m+b$	$3m+b$

m m m m

Algebraically (from definition):

$$\begin{aligned}\frac{\mathrm{d}}{\mathrm{d}x}(mx+b) &= \lim_{h\to 0}\frac{m(x+h)+b-(mx+b)}{h} \\ &= \lim_{h\to 0}\frac{mh}{h} = \lim_{h\to 0} m = m\end{aligned}$$

2. Some derivative properties

$\frac{\mathrm{d}}{\mathrm{d}x}kf(x)$: **The derivative of a constant times a function.**

Numerical intuition: Find the slopes between each pair of points to approximate the derivative of $f(x)$:

x	-3	0	3	6	9
$f(x)$	10	8	1	5	11

Now repeat for the function $y = 5f(x)$:

x	-3	0	3	6	9
$5f(x)$	50	40	5	25	55

How are the slopes of the two tables related?

The proof follows similar logic but uses limit properties and the definition of a derivative function which may make it seem more abstract.
Remember the definition of the derivative function is $f'(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$

Proof:

$$\begin{aligned}\frac{\mathrm{d}}{\mathrm{d}x}kf(x) &= \lim_{h\to 0}\frac{kf(x+h)-kf(x)}{h} \\ &= \lim_{h\to 0} k\frac{f(x+h)-f(x)}{h} \\ &= k\lim_{h\to 0}\frac{f(x+h)-f(x)}{h} \\ &= kf'(x)\end{aligned}$$

□

$\frac{\mathrm{d}}{\mathrm{d}x}(f(x)+g(x))$: **The derivative of the sum of functions.**

Numerical intuition: Find the slopes between each pair of points to approximate the derivatives of $f(x)$ and $g(x)$:

x	-3	0	3	6	9
$f(x)$	10	8	1	5	11

x	-3	0	3	6	9
$g(x)$	-2	1	6	15	27

Now repeat for the function $y = f(x) + g(x)$:

x	-3	0	3	6	9
$f(x)+g(x)$	8	9	7	20	38

How are the slopes of f, g, and $f+g$ related?

The proof follows:

Proof:

$$\begin{aligned} \frac{\mathrm{d}}{\mathrm{d}x}(f(x)+g(x)) &= \lim_{h\to 0}\frac{(f(x+h)+g(x+h))-(f(x)+g(x))}{h} \\ &= \lim_{h\to 0}\frac{f(x+h)-f(x)+g(x+h)-g(x)}{h} \\ &= \lim_{h\to 0}\frac{f(x+h)-f(x)}{h} + \lim_{h\to 0}\frac{g(x+h)-g(x)}{h} \\ &= f'(x)+g'(x) \end{aligned}$$

□

The same argument applies to $\frac{\mathrm{d}}{\mathrm{d}x}(f(x)-g(x))$ so in general we say $\frac{\mathrm{d}}{\mathrm{d}x}(f(x)\pm g(x)) = f'(x)\pm g'(x)$.

3. You might wonder if the same is true of the product of two functions, $\frac{\mathrm{d}}{\mathrm{d}x}(f(x)\cdot g(x))$. As before, it's worth some time to explore intuitively first so reuse the tables above to see if $\frac{\mathrm{d}}{\mathrm{d}x}(f(x)\cdot g(x)) \stackrel{?}{=} f'(x)\cdot g'(x)$.

x	-3	0	3	6	9
$f(x)$	10	8	1	5	11

x	-3	0	3	6	9
$g(x)$	-2	1	6	15	27

x	-3	0	3	6	9
$f(x)\cdot g(x)$	-20	8	6	75	297

What does this suggest is (or isn't) true about $\frac{\mathrm{d}}{\mathrm{d}x}(f(x)\cdot g(x))$?

4. **Power Rule Examples:**

(a) $\frac{\mathrm{d}}{\mathrm{d}x}\left(5x^2\right)$

(b) $\frac{\mathrm{d}}{\mathrm{d}x}\left(\pi x^5 - \frac{2}{x^3}\right)$

(c) $\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{1}{x} - \frac{3}{\sqrt{x}} + \sqrt[5]{x^3}\right)$

(d) $\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{x^4-7x}{x^2}\right)$