Consider the functions \( f(x) = 18 - x^2 \) and \( g(x) = 2x^2 - 9 \) shown below (see Figure 1). What is the area of the largest rectangle that can be inscribed inbetween these functions?

**Answer:** \( 36\sqrt{3} \)

Consider the line tangent to the function \( f(x) = \frac{5}{x^2} \) at the point \( (w, \frac{5}{w^2}) \).

Let \( A(w) \) be the area of the triangle formed by the line tangent to \( f(x) \) at \( (w, \frac{5}{w^2}) \), the \( x \)-axis, and the \( y \)-axis. Find the value of \( w \) that maximizes \( A(w) \) on \([\frac{1}{2}, 3] \). Use calculus to explain this result.

**Answer:** \( t = -\frac{10}{w^2}x + \frac{15}{w} \)
\[
A(w) = \frac{45}{4w^2} \quad A'(w) = -\frac{45}{4w^3} \text{ so c.p. out of domain.}
\]
\[
A\left(\frac{1}{2}\right) = \frac{45}{2} \quad A(3) = \frac{15}{4} \text{ So max at } w = \frac{1}{2}
\]
3. Suppose you are given 100 meters of fence which you must use to form two separate corals; one in the shape of a square, the other in the shape of a circle.

![Circular and Square corrals](image)

(a) What lengths must you cut in order to enclose the maximum total area?

(b) What is the minimum area that can be enclosed and under what circumstances?

Answer: \( A'(x) = \frac{2x(\pi+4)-200}{\pi} \)

c.p. at \( x = \frac{100}{\frac{\pi}{4}} \)

\( A\left(\frac{100}{\frac{\pi}{4}}\right) = 350.06 \text{m}^2 \)

\( A(0) = 795.77, A(25) = 625 \)

4. A building is surrounded by an 8 foot fence placed 4 feet from the building. What is the shortest ladder you need in order to lean it against the building, over the fence?

![Ladder diagram](image)

Answer: Let \( x \) be the length of the base, then for \( x = 10.35, h = 13.04 \) we have \( \ell = 16.65 \) ft.