Math 251

Practice Problems Name:________________

Show all relevant work!

YOU MAY USE A CALCULATOR TO VERIFY SOLUTIONS, BUT NOT TO PROVIDE THEM.

1. Suppose $h(x) = (f(x))^3$. If $f(1) = -2$ and $f'(1) = 5$, find $h'(1)$.

2. The half-life of of Pu-238 is 88 years. Let Q_0 represent the initial quantity of Pu-238 and assume the decay of the element is continuous.

(a) Using base e, write the particular equation giving the quantity, Q, in grams as a function of time, t, in years.

(b) Find $\frac{dQ}{dt}$ and explain what it represents. Include units in your explanation.

(c) What is the annual rate of decay from your model in (a)?
Compare and contrast this rate with your answer to part (b).

3. The daily cost, C, of running an air conditioner in Arizona depends on the temperature, H, as shown in the first table. The temperature in turn increases with the time of day, t, as shown in the second table. Determine the rate at which cost changes with time when $t = 10$ and interpret the result.

H (in F°)	90	95	100	105	110
$C(H)$ (\$)	4	4.75	6	7.50	9.15

t (in hours past 00:00)	6	8	10	12	14
$H(t)$ (in F°)	90	97	100	112	119

4. Suppose $m'(x) = \dfrac{x^3}{\sqrt{1-x^4}}$. What is a possible formula for $m(x)$?

5. Show that $\frac{\mathrm{d}}{\mathrm{d}x} \arccos x = \dfrac{-1}{\sqrt{1-x^2}}$

6. Use the table below to help you find $\frac{\mathrm{d}}{\mathrm{d}x} f^{-1}(x)$ evaluated at $x = 4$.

x	0	1	2	3	4
$f(x)$	1	2	4	5	5.6
$f'(x)$	1	$\frac{9}{5}$	$\frac{3}{2}$	$\frac{4}{5}$	$\frac{1}{3}$

7. Assuming we already know $\frac{\mathrm{d}}{\mathrm{d}x}(e^x) = e^x$, show $\frac{\mathrm{d}}{\mathrm{d}x}(b^x) = \ln(b) b^x$ for $b \in \mathbb{R}^+$.

8. Assuming we already know $\frac{\mathrm{d}}{\mathrm{d}x}(\ln x) = 1/x$, find $\frac{\mathrm{d}}{\mathrm{d}x}(\log_b x)$ (for $b \in \mathbb{R}^+$).

9. Where is $y = \arctan(1 - x^2)$ increasing and where is it decreasing? Where is it concave up?