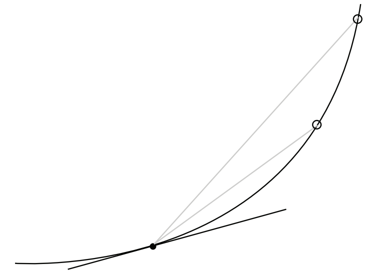
Math 251

Fall 2016 M – Th 12:40 – 1:45 (Section AX/BH)



Instructor: Jon Freedman
Office: 7216
Phone: 738 – 7032
e-mail: freedmanj@smccd.edu
Website: www.smccd.edu/accounts/freedmanj/

Office Hours: M – Th 1:45 – 2:30; TuTh 10 – 11 and most times by arrangement – *ask*.

Prerequisite: Math 222 with C or better, or appropriate score on placement test.

Important Details: (1) Math 251 is a prerequisite for majors in Engineering, Mathematics, and Physics. Check yours.

(2) Transfer: CSU (B4); UC (credit limit).

Text: Hughes-Hallett, Gleason, McCallum, et al. *Single Variable Calculus.* 6th ed. New York: John Wiley & Sons Inc. 2013. (yellow binding). To go through Math 253 (multivariate calculus), buy the red bound book, *Single and Multivariable Calculus.* Packaged with WileyPlus. OR you can get it as an e–book.

Materials: A TI–84 (or TI–83+) graphing calculator is required for this course. Other graphing calculators may perform the same functions and may be acceptable but see me about this. If you have a TI-89, TI-92, N-Spire, or other technology that can perform symbolic manipulations you may not be allowed to use it on some forms of assessment.

Important Dates:

Last day to Add this course:	Tuesday, August 30
Last day to Drop this course without a W:	Monday, September 5
Last day to Withdraw from class:	Wednesday, November 16
Holidays:	9/5; 11/11; 11/24 – 27
Last regular class:	Thursday, December 8
Final Exam (comprehensive):	Monday, Dec. 12 11:10 – 1:40 pm

Assignments: Regular assignments will be given in class (handouts), online through WileyPlus (packaged with the textbook or available online) or assigned from the book. There will be weekly (often open note) quizzes based on class and homework assignments. There will be a written project concurrent with chapter 4. .

Grading: Assignments (homework, TBA, classwork, quizzes) (30%)

3 – 5 Tests (50%)

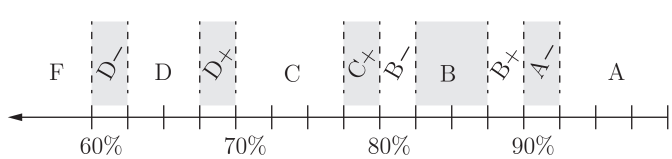
Final (20%)

You will also be required to pass a techniques test in order to pass the class. You may retake the test as often as you wish but you must pass it with at least 95% proficiency. If you pass the methods test before November 24 you may earn 10/10 test points towards your grade.

I will drop your worst test score (Not the final). There will be no makeup tests. If you are late for a test you will have only the remaining time to complete the test (so don't be late). If you know you are going to miss a test date, contact me at least three days in advance and we can arrange an alternate test to be taken in *advance* of the class test date.

I will excuse your worst score from each category (HW, tests, quizzes) but will not give makeup work. I will give quizzes often. I will drop your worst quiz. There will be no makeup quizzes.

Grading Scale:



Attendance: Your involvement in class and your participation in the process of discovering concepts will be fundamental in your understanding of calculus. I try not to lecture directly from the book but rather to provide experiences enhanced by the book. If you are habitually late or absent from class it is unlikely that you will pass the course.

Withdrawal Policy: If you decide to drop this class you must do so formally either by using WebSMART or by filing the correct form with the registrar's office. The likelihood of you passing the class after eight absences is almost 0. If you miss more than 10 hours of class and still desire to remain in the class you must meet with me and convince me that you can learn the material necessary to pass the class.

Course Contents: We will cover the majority of Chapters 1 through 5, as well as some supplemental materials. By the completion of the course you will be able to demonstrate the following skills:

- For each of the functions:
  - Linear
  - Quadratic (and some general polynomials)
  - Power
  - Trigonometric and Inverse Trig.
  - Exponential
  - Logarithmic
  - Logistic

  you will be able to take any one of the representations: Data Table, Graph, Formula, and use technology or algebraic manipulation to generate the other two.
- Additionally, you will recognize and derive the relationship between each function and Contextual Applications relevant to that function.
- You will be able to use the graphing calculator to help you analyze complex functions, derive formulas from data, and perform various calculusrelated analyses on a function.
- You will understand and anticipate the behavior of functions through translation and distortion.
- You will demonstrate your understanding of the derivative through (correct) numerical approximation, graphical means, symbolic manipulation, and contextual application.
- You will understand the relationship between limits, continuity, and differentiability.
- You will be able to apply the derivative in situations involving local linearity (e.g. L'hospital's Rule), and implicit differentiation (e.g. inverse function derivatives).
- You will solve application problems involving graphing, related rates, and optimization targeting the fields of business, life science, and physics. You will be able to reason the correct calculusbased or algebraic means of solving problems in these and related subjects.
- You will demonstrate your understanding of the definite integral through (correct) numerical approximation, contextual application and the Fundamental Theorem of Calculus.
- You will demonstrate your understanding of the antiderivative through graphical means, and symbolic manipulation.

Tutoring: Think seriously about joining MESA (Rm. 7309). If you have any interest in Mathematics, Engineering, or the Sciences you should join MESA and make use of their many support resources. Just sitting in a supportive environment can be a tremendous help.

The Learning Center (TLC) is a good resource for semi-free tutoring in all of your classes.

Assistance: In Coordination with the DRC office, reasonable accommodations will be provided for qualified students with disabilities. If you have an accommodation letter, please meet with me during my office hours to discuss your needs. For more information, please contact DSPS office in building 2 at 738-4280.

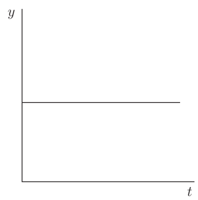
Academic Dishonesty: I strongly encourage you to form study groups and to work together to understand the material covered in this class. Explaining a concept is a valuable way for you and another to develop your insight and your skills. Copying work is of no value to you academically. Consequently, if I find that you are submitting any part of another's work as your own, you will not receive credit for it and it will not be dropped as a lowest score. The same holds true for any other kind of academic dishonesty. There is no situation that could arise in this class that would justify risking expulsion. If you are having any difficulty, PLEASE see me about it so that we can work together in resolving the issue.

1.1 Linear Functions

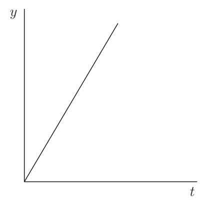
Show all relevant work!

1. Match each graph to the situation it best describes.

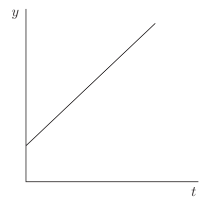
(i)



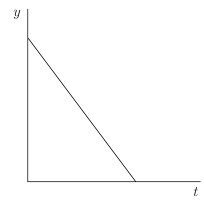
(ii)



(iii)



(iv)



(a) A bathtub filled with 40 gallons of water is draining at 0.8 gallons per minute until it is empty. The volume of water in the tub is plotted against time.

(b) An elevator starts at the second floor, 20 feet above the ground level, and then rises at 2 feet per second. The height of the elevator is plotted against time.

(c) Larry parks his car 15 feet from his house and stays home for the weekend. The distance Larry's car travels from his house is plotted against time.

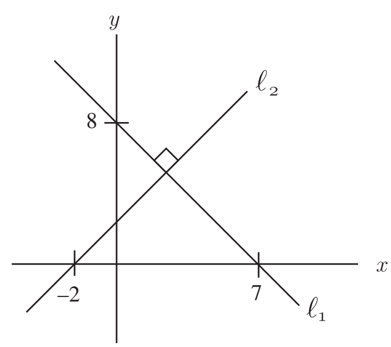
(d) An employee earns $12 per hour. Her income is plotted against time.

2. Write an equation for each situation described in (#1).

(a)______________________________ (b)______________________________

(c)______________________________ (d)______________________________

3. Find the equation of the line ℓ_2.



4. Using algebra, determine the point of intersection for lines ℓ_1 and ℓ_2 in problem (1) above.

5. The table for a linear equation is started below.
(a) Fill in the rest of the table.

(b) What is the equation of the line? ______________________

x		-4	2		
y		7	3		

6. Which of the tables below could represent a linear function?

(a)

t	1	2	3	4	5
$f(t)$	5	5	5	5	5

(b)

x	9	8	7	6	5
$g(x)$	12	16	20	24	28

(c)

t	5	8	12	17	23
$h(t)$	3	5	7	9	11

(d)

x	3	3	3	3	3
$k(x)$	4	2	0	-2	-4

(e)

t	-5	1	7	13	19
$m(t)$	-2	1	-2	1	-2

(f)

t	-6	0	3	9	21
$p(t)$	-8	-4	-2	2	10

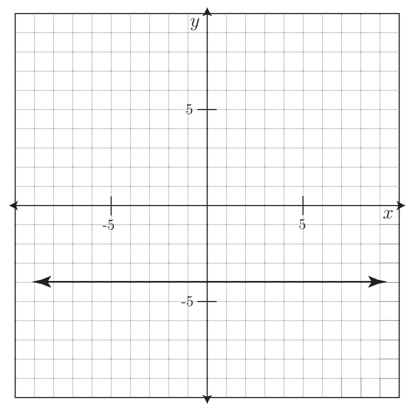
7. Write the corresponding equation for each of the linear functions in #6.

8. Write the equations of the lines graphed below.

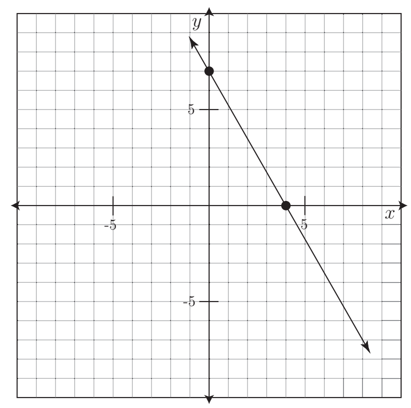
(a) ____________________

(b) ____________________

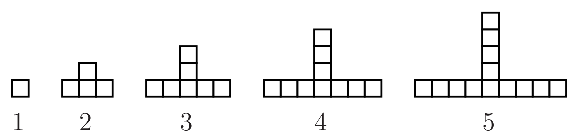
(a)



(b)



9. How many square tiles will it take to make the 3709^{th} figure in the sequence shown to the right?



10. A tub drains so that after 5 minutes 30 gallons of water remain in the tub and after 20 minutes, 21 gallons remain.

(a) If the amount of water in the tub is linearly dependent on the time since the drain was opened, write the equation of the volume of water in the tub, V, as a function of time, t.

(b) What does the slope of this line tell you?

(c) How much water is there in the tub when it begins draining?

(d) Find the t–intercept and interpret its meaning in this context.

1.2 Exponential Problems

Show all relevant work!

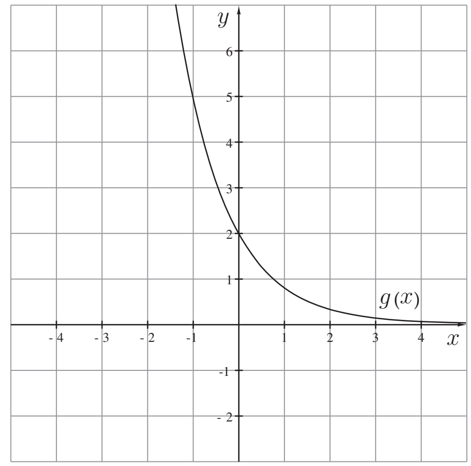
1. Use the exponential function, $g(x)$, graphed below to answer the given questions.

(a) Estimate $g(2)$.

(b) Estimate $g^{-1}(5)$.

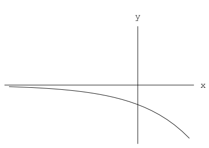
(c) Estimate the solution to $g(x) = x$.

(d) Find the equation for $g(x)$.

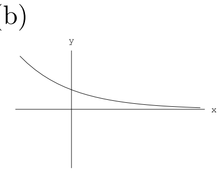


2. Match each of the graphs of $y = a \cdot b^x$ with the description best suited to it. (Find one description for each graph)

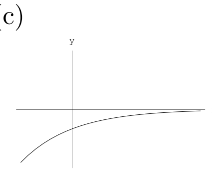
(a)



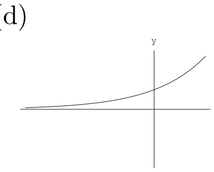
(b)



(c)

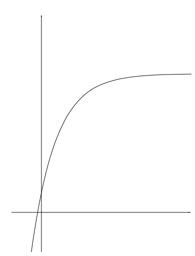


(d)



(i) $a < 0$ and $0 < b < 1$. (ii) $a > 0$ and $0 < b < 1$ (iii) $a < 0$ and $b > 1$ (iv) $a > 0$ and $b > 1$

3. The function below is of the form $y = a \cdot b^x + c$ (what does c do?). Find values for a, b, and c that will produce a graph like the one shown.



Over

4. A tale of two cities.

Suppose $P_1(t) = 1.3 + 0.05t$ and $P_2(t) = 1.3(1.05)^t$ are population models of two different cities giving population in millions as a function of time in years since 2000. Interpret the meaning of the two models in terms of starting population and growth rate.

5. The tables below have function formulas of the form $y = a \cdot b^x$, $y = a \cdot b^x + c$, or $y = ax^2 + bx + c$. For each table, determine its formula.

(a)

t	1	2	3	4	5
$f(t)$	24	36	54	81	121.5

(b)

t	1	2	3	4	5
$g(t)$	0	66	211.2	530.64	1233.408

(c)

t	1	2	3	4	5
$h(t)$	6.5	8	12.5	20	30.5

6. Solve these equations.

(a) $5e^{3x} = 38$

(b) $3(1.015)^{12t} = 11$

7. Describe a situation that might be modeled by #6(b), including an explanation for each of the numbers in the equation.

8. Eric invests \$2500 at 4.7% compounded quarterly.

(a) What will be the balance of Eric's account after 3 years?

(b) How long will it be before the balance of Eric's account reaches \$7,000?
(Round your answer to three decimal places.)

9. Suppose you have a 3.200g sample of Polonium-210 and you observe that over a period of 30 days it decays to 2.752g. Determine the half-life of Polonium–210.

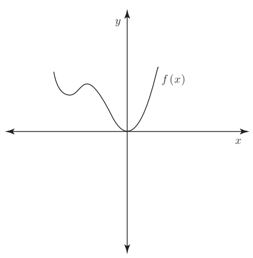
Some Investigations

0.1 Shifts and Stretches

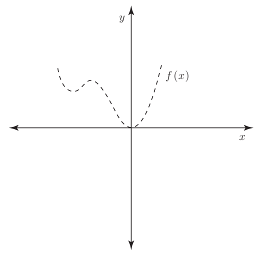
Recall the various linear (multiplication or addition by a constant) manipulations of a function and their related effects. Complete the sketches and tables for each manipulation.

Shifts (vertical)

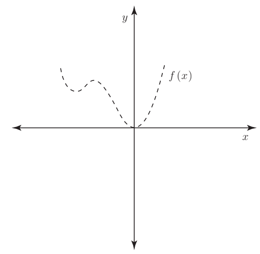
graphic:



$f(x)$



$f(x) + a$
$(a > 0)$



$f(x) - a$
$(a > 0)$

tabular:

x	-2	-1	0	1	2
$f(x)$	4	2	0	2	5

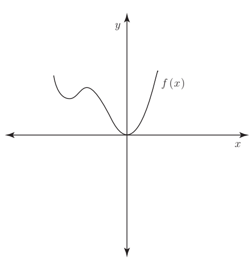
x	-2	-1	0	1	2
$f(x) + a$					

x	-2	-1	0	1	2
$f(x) - a$					

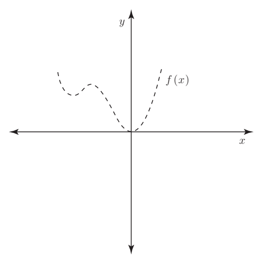
(e.g. $a = 2$)

Shifts (horizontal)

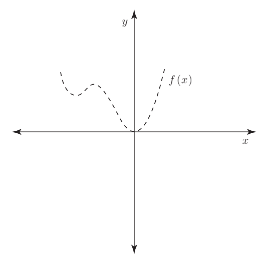
graphic:



$f(x)$



$f(x + a)$
$(a > 0)$



$f(x - a)$
$(a > 0)$

tabular:

x	-2	-1	0	1	2
$f(x)$	4	2	0	2	5

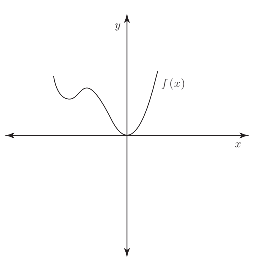
x	-2	-1	0	1	2
$f(x + a)$					

x	-2	-1	0	1	2
$f(x - a)$					

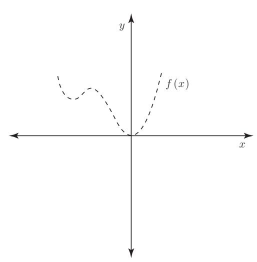
(e.g. $a = 2$)

Reflections (vertical) (horizontal)

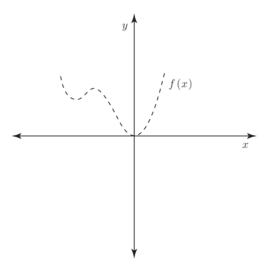
graphic:



$f(x)$



$-f(x)$



$f(-x)$

tabular:

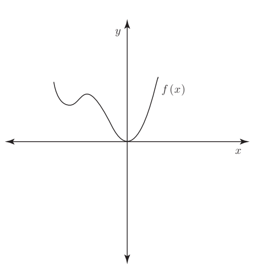
x	-2	-1	0	1	2
$f(x)$	4	2	0	2	5

x	-2	-1	0	1	2
$-f(x)$					

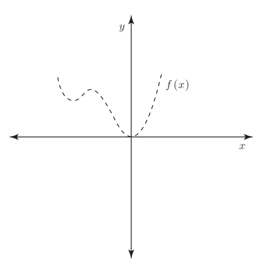
x	-2	-1	0	1	2
$f(-x)$					

Stretches/Compressions (vertical)

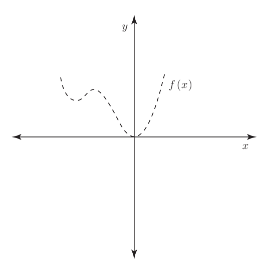
graphic:



$f(x)$



$af(x)$
$(a > 1)$



$af(x)$
$(0 < a < 1)$

tabular:

x	-2	-1	0	1	2
$f(x)$	4	2	0	2	5

x	-2	-1	0	1	2
$af(x)$					

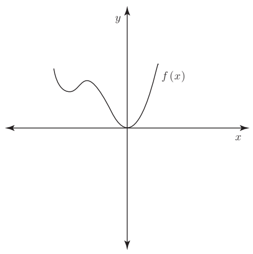
(e.g. $a = 2$)

x	-2	-1	0	1	2
$af(x)$					

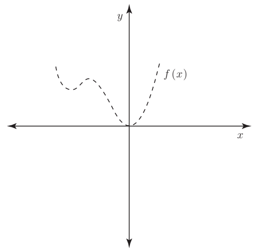
(e.g. $a = \frac{1}{2}$)

Stretches/Compressions (horizontal)

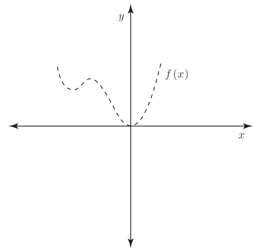
graphic:



$f(x)$



$f(ax)$
$(a > 1)$



$f(ax)$
$(0 < a < 1)$

tabular:

x	-2	-1	0	1	2
$f(x)$	4	2	0	2	5

x	-2	-1	0	1	2
$f(ax)$					

(e.g. $a = 2$)

x	-2	-1	0	1	2
$f(ax)$					

(e.g. $a = \frac{1}{2}$)

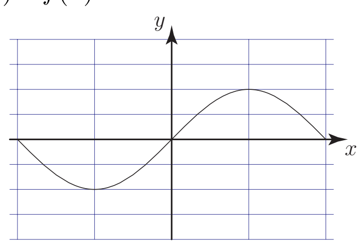
In each of the preceding cases, discuss the effects of the shift or stretch on the average rate of change of the function on the interval $[-2, 2]$.

0.2 Odd & Even Functions

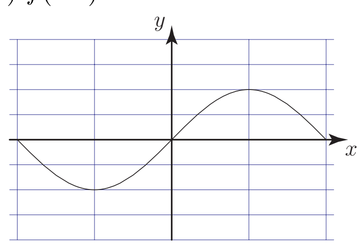
Sketch the following functions.

1. Sketch the indicated reflection over the graph of $g(x)$ in each exercise.

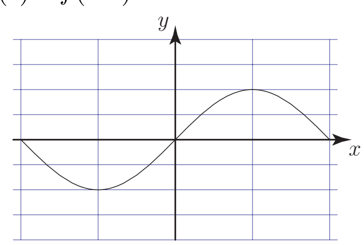
(a) $-f(x)$



(b) $f(-x)$

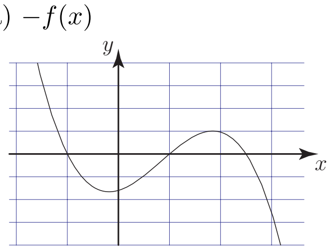


(c) $-f(-x)$

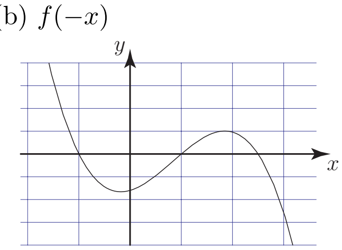


2. Sketch the indicated function over the graph of $f(x)$ in each exercise.

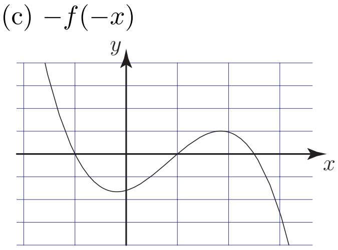
(a) $-f(x)$



(b) $f(-x)$

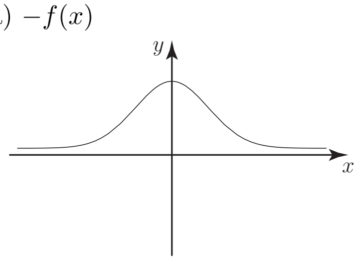


(c) $-f(-x)$

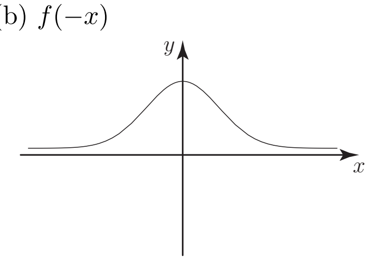


3. Sketch the indicated function over the graph of $f(x)$ in each exercise.

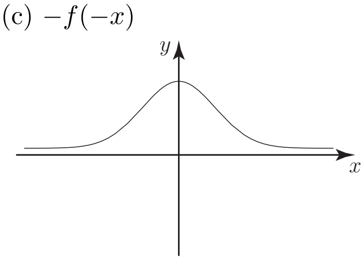
(a) $-f(x)$



(b) $f(-x)$



(c) $-f(-x)$



Let's remember the definitions of odd and even functions:

Definition 0.1 *Odd and Even*

A function, $f(x)$, is even if $f(-x) = f(x)$ for any x in its domain.
A function, $f(x)$, is odd if $f(-x) = -f(x)$ (or equivalently $f(x) = -f(-x)$) for any x in its domain.

Which of the functions above are even? odd? neither? Explain.

Do odd and even functions behave like odd and even numbers? (How far does the metaphor extend?)
For example, we know the sum of two even numbers is an even number, is the sum of two even functions an even function? What about two odd functions? An odd and an even? What about the rules for products?

Begin your inquiry by choosing some examples and playing around with them. Remember that in order to disprove something you only need a single example showing it's not true. However, in order to *prove* something, you need to work in general terms using definitions rather than specific cases to show it works for *all* cases.

Example 0.1

The sum of an even function and an even function is an even function:
We want to show that if f and g are even functions, then their sum is an even function.
Let $h(x) = f(x) + g(x)$. We need to show that $h(-x) = h(x)$.
$h(-x) = f(-x) + g(-x)$
$= f(x) + g(x)$ Since both f and g are even.
$= h(x).$ Notice that we started with $h(-x)$ and concluded with $h(x)$.
Therefore the sum of two even functions is even.

Questions 0.1

1. Sums. Give either a proof or a counter example for each.
   (a) Is the sum of an odd function and an odd function even? Odd? Neither?
   (b) Is the sum of an odd function and an even function odd? Even? Neither?

2. Products. Give either a proof or a counter example for each.
   (a) Is the product of an even function and an even function even? Odd? Neither?
   (b) Is the product of an odd function and an odd function odd? Even? Neither?
   (c) Is the product of an odd function and an even function even? Odd? Neither?

0.3 Compositions

1. If $h(x) = f(g(x))$, decompose each function into functions f and g where $f(x) \neq x$ and $g(x) \neq x$.

(a) $h(x) = \sin(x^2 + 1)$
$f(x) =$ ____________ $g(x) =$ ____________

(b) $h(x) = (3x - 2)^3$
$f(x) =$ ____________ $g(x) =$ ____________

(c) $h(x) = \frac{1}{x^2-4}$
$f(x) =$ ____________ $g(x) =$ ____________

(d) $h(x) = e^{-x^2}$
$f(x) =$ ____________ $g(x) =$ ____________

(e) $h(x) = \frac{1}{\sqrt{3-x}}$
$f(x) =$ ____________ $g(x) =$ ____________

(f) $h(x) = \frac{1}{x^2} - 4$
$f(x) =$ ____________ $g(x) =$ ____________

2. What are the domains of the following functions?

(a) $a(x) = \ln x$ (b) $b(x) = \frac{1}{x^2+1}$ (c) $c(x) = a(b(x))$ (d) $d(x) = b(a(x))$

3. (a) On what interval in $[-\pi, \pi]$ is the function $s(x) = x^2$ increasing? On what interval is it decreasing?

(b) On what interval(s) in $[-\pi, \pi]$ is the function $t(x) = \sin x$ increasing? Decreasing?

(c) On what interval(s) in $[-\pi, \pi]$ is the function $w(x) = t(s(x))$ increasing? Decreasing?

4. The tables of $m(x)$ and $p(x)$ are given below.

x	-6	-4	-2	0	2	4	6
$m(x)$	-1	5	8	7	3	-2	-11

x	-7	-1	5	8	7	3	-2
$p(x)$	-6	-4	-2	0	2	4	6

If $u(x) = p(m(x))$, find . . .

(a) $u(0) =$ ________________

(b) $u(2) =$ ________________

If $w(x) = m(p(x))$, find . . .

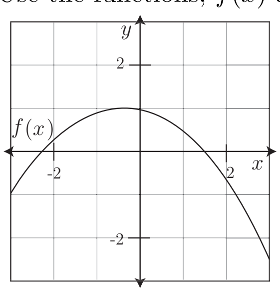
(c) $w(-1) =$ ________________

(d) $w(2) =$ ________________

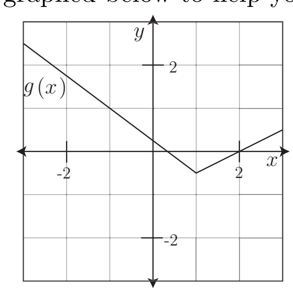
(e) For what value of x is $w(x) = 7$?

5. Suppose $f(x)$ is an even function and $g(x)$ is an odd function. If $h(x) = f(g(x))$ and $k(x) = g(f(x))$, comment on whether they are even, odd, or neither.

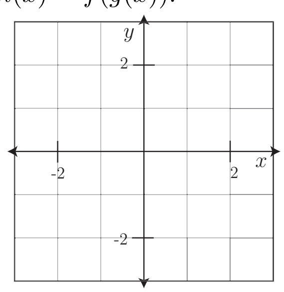
6. Use the functions, $f(x)$ and $g(x)$ graphed below to help you sketch $h(x) = f(g(x))$.



$f(x)$

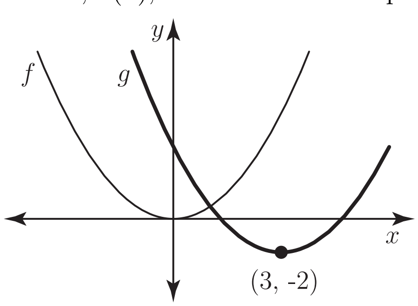


$g(x)$



$h(x)$

7. $g(x)$ is a translation of the parabola $f(x)$ as shown below. Write functions for vertical, $v(x)$, and horizontal, $h(x)$, translations and express $g(x)$ as a composition in terms of f, h, and v.



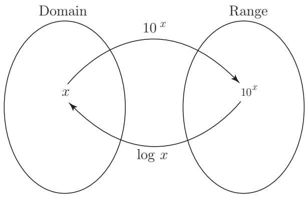
$h(x) =$________________

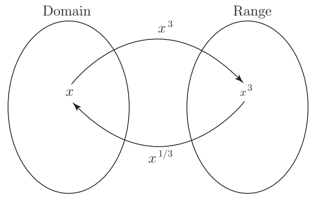
$v(x) =$________________

$g(x) =$________________

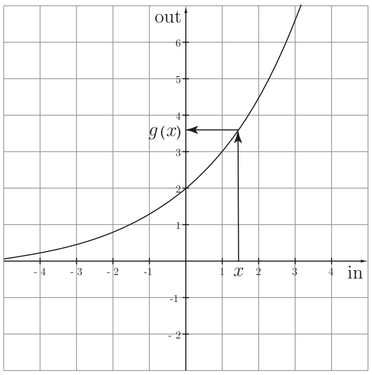
0.4 Inverses

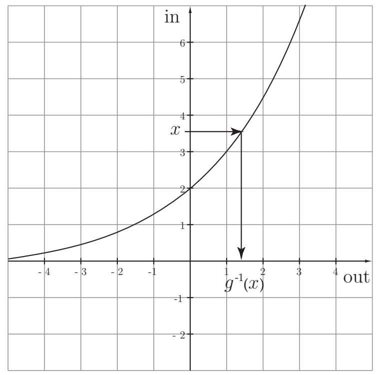
If a function maps values in its domain to new values in its range, then its inverse reverses the process by mapping values in the range (of the original function) back to its domain. (see below).





Equivalently,





In order for a function to have an inverse it isn't enough that we define a mapping reversing the original process. In order to actually reverse the original function, we must have some guarantee that we will get back to where we started from, that is, the inverse must also be a function. For this reason, $f(x) = x^2$, $x \in \mathbb{R}$ does not have an inverse (since $f^{-1}(x) = \pm\sqrt{x}$ and $f^{-1}(9)$ could map to either -3 or 3, for example.), however, $f(x) = x^2$, $x \in \mathbb{R}^+$ does have an inverse, namely $f^{-1}(x) = \sqrt{x}$.

1. Are the functions describing these situations invertible?

   (a) The temperature of an oven as a function of the time since it was turned on.

   (b) The number of people on a bus as a function of the time of day.

2. For $g(x) = 10^x - x$, find $g^{-1}(997)$.

3. For each function below, identify a proper domain and give the inverse function.

   a) $f(x) = e^{1-3x}$ b) $g(x) = x^2 - 1$ c) $h(x) = \dfrac{x}{x+1}$

4. The following table shows values for the function f.

x	-6	-3	-1	0	1	4	7
$f(x)$	-6	-5	0	2	4	7	9

   (a) Find $f(4)$.

   (b) Solve $f(x) = 4$.

   (c) Solve $f(x) = x$.

   (d) Find $f^{-1}(9)$.

   (e) Solve $f^{-1}(x) = 0$.

5. Use the exponential function, $g(x)$, graphed below to answer the given questions.

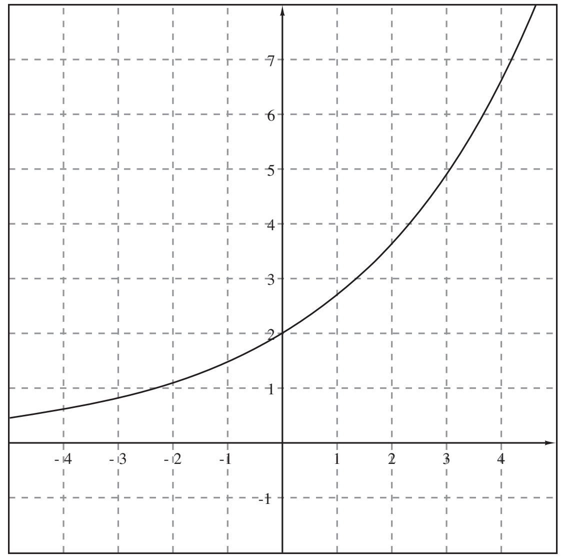
(a) Estimate $g(0)$.

(b) Estimate $g(2)$.

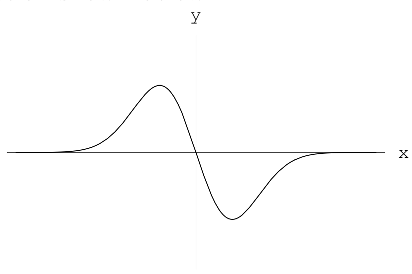
(c) Estimate $g^{-1}(1)$.

(d) Estimate $g^{-1}(5)$.

(e) Estimate $g^{-1}(5.5)$.



6. On the same set of axes, sketch the inverse of the function shown below.



7. If $g(x) = 2^x + \sqrt{x} - 1$, find $g^{-1}(17)$.

8. Suppose $P = f(t)$ gives the population of a city in millions of people as a function of time in years since 1950.
(a) Give the units and interpret the meaning of $f(30) = 9.2$.

(b) Give the units and interpret the meaning of $f^{-1}(15) = 49$.

1.4 Still More Exponential (log) Problems

Show all relevant work!

1. Katrina buys a new plasma for \$3200 but doesn't have to pay for one year. While she waits, however, finance charges accrue at 6% compounded continuously. If Katrina just misses the pay off date at the end of one year and has to pay the interest as well as the original cost, how much will she owe at the end of the year? What is the equivalent APR for this loan?

2. Write an exponential equation for the function containing $(-3, 10)$ and $(2, 2)$.
   (a) Using the form $y = a \cdot b^x$
   (b) Using the form $P = P_0 e^{rt}$

3. Air pressure in the Earth's atmosphere decreases exponentially as a function of altitude above the Earth's surface. Air pressure at sea level (the surface of the Earth) is approximately 14.7 pounds per square inch (psi), while the pressure at 2000 feet is about 13.5 psi.

   (a) Write a formula giving air pressure, P, as a function of altitude, h.

   (b) Find the air pressure at each of the elevations below.
   - ⋄ Mexico City (7500 feet)
   - ⋄ Mount Everest (29,000 feet)
   - ⋄ The edge of space (defined by NASA to be 50 miles above the planet).

   (c) Human blood (at body temperature) will boil if the pressure is below 0.9 psi. At what altitude would your blood start to boil if you were in an unpressurized airplane?

4. The balance of account A is given by the formula $P = 2000(1 + .05)^t$. The balance of account B is given by $P = 2000e^{.05t}$.
   (a) Sketch the two graphs on the same axes and comment on any points the two graphs share in common.

   (b) Which graph appears to grow faster?

   (c) Using numbers and any relevant computations, explain why (and how) one function grows more quickly than the other.

5. The balance of account A (in thousands) is given as a function of time (in years) by $P = 0.31t + 4.7$.
The balance of account B (in thousands) is given as a function of time (in years) by $P = 4.7e^{.31t}$.
Compare the two accounts - what do they have in common and how are they different?
Include details on how they grow (be specific) and the significance of the various numbers in the equations.

6. Solve the equations below.

   (a) $\log(2x) = -5.4$
   (b) $3 \cdot 5^x = 17 \cdot 4^x$

7. Enter $\log(-1)$ in your calculator and explain why you get the result you get.

8. Recall that pH tells us the concentration of hydronium ions (H_3O^+) per liter of the substance. For example, in the case of a pH of 5.4, the concentration of $[H_3O^+]$ is $10^{-5.4} \times (6.022 \times 10^{23}) = 2.397 \times 10^{18}$ molecules per liter.
   What is the concentration of hydronium ions in a solution with a pH of 9.7?

1.5 Trigonometry Problems

Show all relevant work!

1. If a fireman leans a 24 foot ladder against a building at a 70° angle,

(a) How far from the building is the base of the ladder?

(b) How high (above the ground) does the ladder reach on the building?

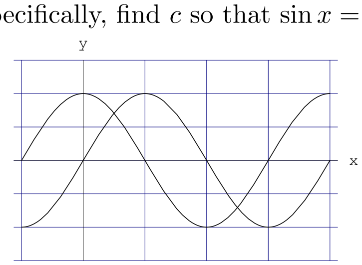
2. A merry-go-round with a 10 ft. diameter is spinning at 12rpm.

(a) What is the angular velocity of the merry-go-round in radians per second?

(b) How fast (in feet per second) is Raul travelling if he sits on the outer edge of the merry-go-round?

(c) How fast is Klaus travelling (in feet per second) if he sits at the center of the merry-go-round?

3. The functions $\sin x$ and $\cos x$ are almost identical except for horizontal position (see below). That means you should be able to express $\sin x$ as a shift of $\cos x$ and similarly, $\cos x$ as a shift of $\sin x$. Specifically, find c so that $\sin x = \cos(x + c)$ and find c so that $\cos x = \sin(x + c)$.



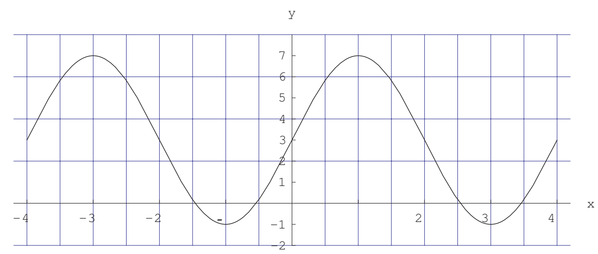
4. Solve the following for $x \in \mathbb{R}$.

(a) $\sin x = 1$

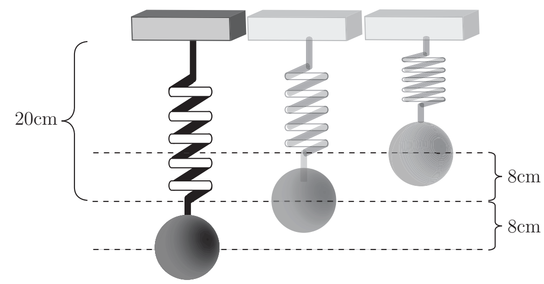
(b) $\cos 3x = 1$

(c) $\sin(x^2 - 1) = 1$

5. Find two different equations for the periodic function shown on the right. (One in terms of $\sin x$ and the other in terms of $\cos x$).



6. A mass is suspended at the end of a spring where it hangs 20cm from the ceiling. It is displaced 8cm below its rest position and released. It reaches the point closest to the ceiling (12cm) after 1 second. Write a periodic model for this situation giving the distance of the mass from the ceiling, y, as a function of time, t.



7. The table below shows the US average unemployment rate at the beginning of each year from 2006 – 2016. (Where 2006 is $t = 0$).

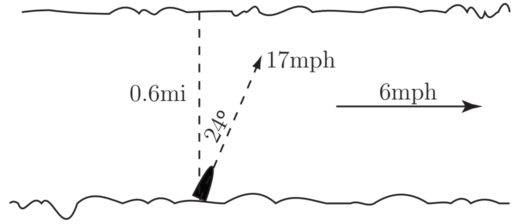
t (years)	0	1	2	3	4	5	6	7	8	9	10
$U(t)$ (% unemployment)	4.6	4.7	5.0	7.8	9.8	9.1	8.3	8.0	6.6	5.7	4.9

a) Write a periodic function that models these data using methods discussed in class.

b) Use your model to predict the unemployment rate for January of 2020.

8. A boat launches from one shore of a river at a heading 24° downstream. The river is 0.6 miles wide in most places. If the current moves at 6mph and the boat's speed, relative to the water, is 17mph,

(a) How far down the opposite shore will the boat arrive?



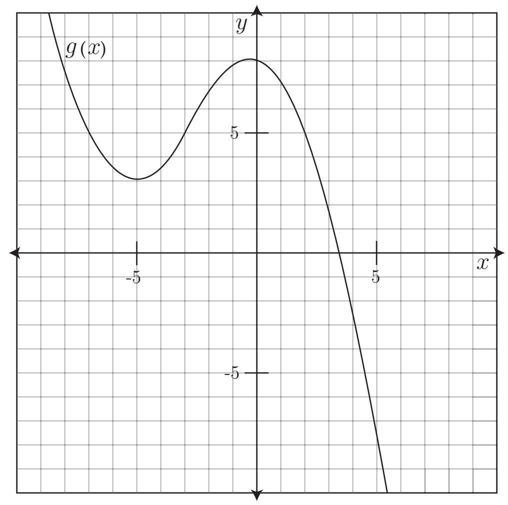
(b) How far will the boat have traveled getting there?

(c) How fast (relative to the shore) did the boat travel?

(d) How long will it take to get there?

1. Find the x–intercepts of $f(x) = \cos(x^2 - 1)$ for $x \in \mathbb{R}$.

2. Determine a reasonable equation for $g(x)$ below.



3. Complete the table below if f is linear, g is quadratic, and h is exponential.

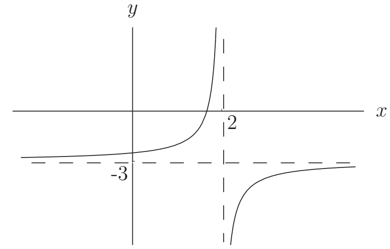
t	-3	-2	-1	0	1	2	3
$f(t)$	7	3					
$g(t)$	7	3	-5				
$h(t)$	7	3					

4. A person's weight varies (approximately) with the cube of their height. If Larry is $5'8''$ and weighs 160 pounds, how much would Aaron, who is $6'2''$, weigh assuming his build is of similar proportion.

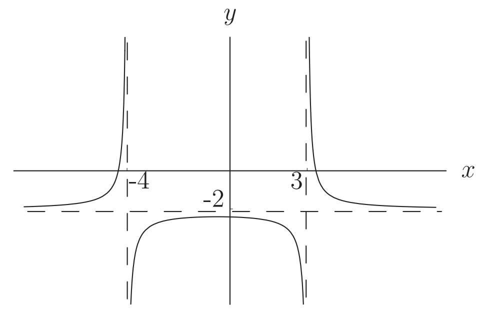
5. The base diameter of a tree (measured in cm) varies directly with the $\frac{3}{2}$ power of its height (measured in meters).
   (a) If a tree 5 meters high has a base diameter of 14.5 cm, find the constant of proportionality and write a function relating the height and base diameter of any similar trees.

   (b) Find the height of a tree with a base diameter of 238 cm.

6. The graph below is a translation of the function $f(x) = \dfrac{1}{x}$.
   Find a possible formula for the graph and apply algebra to write it as the ratio of two linear functions.



7. Give a possible equation for the function graphed below.



8. Determine the zeros, y-intercept, and asymptotes (vertical and horizontal) for the function below. Then sketch it.

$$g(x) = \frac{x^2 - 25}{x^2 - 4x - 12}$$

9. Suppose $f(1) = 5$ and $f(5) = 16$. Find a possible formula for f if f is

(a) linear

(b) exponential

(c) power function

10. Find $f^{-1}(x)$ for $f(x) = \dfrac{2x - 1}{5 - 3x}$.

1.7 Continuity

Show all relevant work!

YOU MAY USE A CALCULATOR TO VERIFY SOLUTIONS, BUT NOT TO PROVIDE THEM.

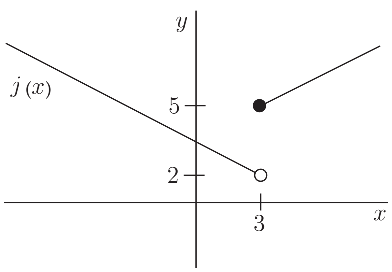
1. (a) Is the function $g(x) = \dfrac{e^{-x}}{\cos x}$ continuous on $[0, \pi]$?

(b) Is $k(x) = \sqrt{1 + x^2}$ continuous on $(-\infty, \infty)$?

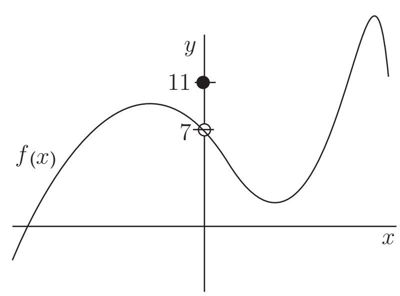
2. Find the value for k that will make $f(x)$ continuous.

$$f(x) = \begin{cases} kx^2 - 5 & : \quad x \leq 2 \\ 3x + 4 & : \quad x > 2 \end{cases}$$

3. Find the limit $\lim\limits_{x \to 0} f(x)$ (if it exists).



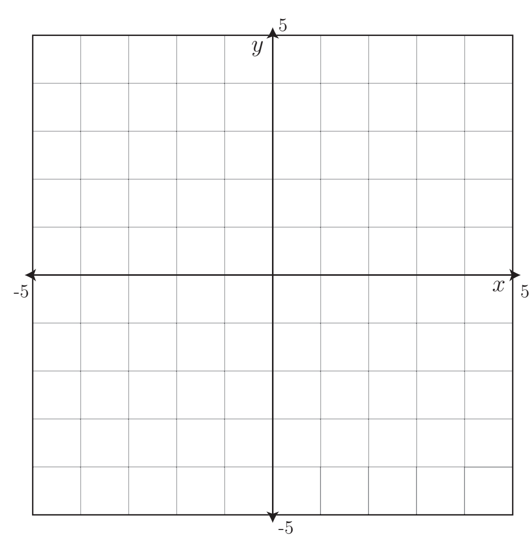
4. Find the limit $\lim\limits_{x \to 3} j(x)$ (if it exists).



5. Find the limit $\lim\limits_{x \to 0} k(x)$

$$k(x) = \begin{cases} \cos x & : \quad x \leq 0 \\ 2 - x & : \quad x > 0 \end{cases}$$

Sketch the graph to confirm your answer.



6. Estimate $\lim_{x\to 0} \dfrac{\sin x}{x}$ (make a table of values).

7. A rocket is launched from the ground with an initial velocity of 140 ft/sec. Assuming it travels straight up and ignoring air resistance (or any other kind of reality), answer the following questions:

(a) Write an equation for the height of the rocket, h, as a function of time, t.

(b) Find $h(2)$ and interpret the meaning of your result.

(c) When does the rocket return to the earth?

(d) What is the highest altitude the rocket reaches and how long does it take to get there?

8. What is the minimum initial velocity needed to propel a rocket to an altitude of 500 feet?

9. A bullet is shot straight up and returns to the ground 34 seconds later. What was the initial velocity of the bullet?

Limits and Other Problems

Show all relevant work!

YOU MAY USE A CALCULATOR TO VERIFY SOLUTIONS, BUT NOT TO PROVIDE THEM.

1. Using figures 1 and 2, estimate

(a) $\lim_{x\to 1^-} (f(x)+g(x))$: ____________

(b) $\lim_{x\to 1^+} (f(x)+2g(x))$: ____________

(c) $\lim_{x\to 1^-} f(x)g(x)$: ____________

(d) $\lim_{x\to 1^+} \dfrac{f(x)}{g(x)}$: ____________

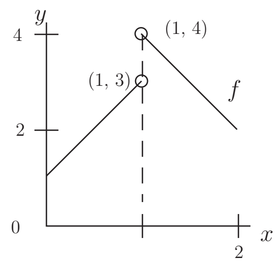


Figure 1

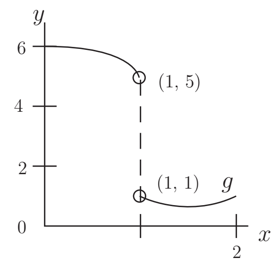


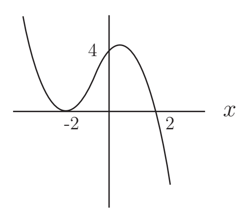
Figure 2

2. Investigate $\lim_{h\to 0}(1-h)^{1/h}$ numerically (make a table of values).

3. Find a value of the constant k such that the limit exists.

$$\lim_{x\to 1} \frac{x^2-kx+4}{x-1}$$

4 Find a cubic polynomial for the graph below.



4. The height of an object above ground at time t is given by $s = v_0 t - \frac{g}{2}t^2$, where v_0 is the initial velocity and g is acceleration due to gravity.

(a) At what height is the object initially?

(b) How long is the object in the air before it hits the ground?

(c) When will the object reach its maximum height?

(d) What is that maximum height?