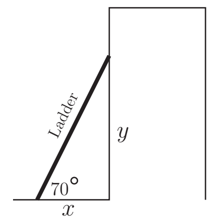
Trigonometry Problems **Solutions**

1. If a fireman leans a 24 foot ladder against a building at a $70°$ angle,



(a) How far from the building is the base of the ladder?

Solution: We want the distance from the building (labeled x). We know the length of the ladder is $24'$ so relative to the $70°$ angle we **know** the hypotenuse and we **want** the side adjacent so we use cosine:

$\cos 70° = \frac{x}{24} \longrightarrow 24\cos 70° \approx 8.2'$ $\square$

(b) How high (above the ground) does the ladder reach on the building?

Solution: We want the distance from the building (labeled y). We know the length of the ladder is $24'$ so relative to the $70°$ angle we **know** the hypotenuse and we **want** the side opposite so we use sine:

$\sin 70° = \frac{y}{24} \longrightarrow 24\sin 70° \approx 22.6'$ $\square$

2. A merry-go-round with a 10 ft. diameter is spinning at 12rpm.

(a) What is the angular velocity of the merry-go-round in radians per second?

Solution: Since there are 2π radians in one full revolution, the merry-go-round is rotating at $12 \cdot 2\pi = 24\pi$ radians per minute. Then this is the same as $24\pi\frac{\text{rad}}{\text{minute}} \cdot \frac{1\text{ minute}}{60\text{ seconds}} = \frac{2\pi}{5}$ rad per second. $\square$

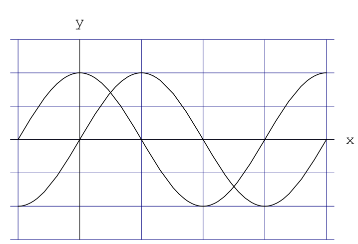
(b) How fast (in feet per second) is Raul traveling if he sits on the outer edge of the merry-go-round?

Solution: Since speed is measured in distance traveled over time, we need to know how far Raul travels in a second or minute. Since he makes 12 full revolutions in a minute and the radius if the merry-go-round is 5ft, he's covering $12 \cdot 2\pi(5) = 120\pi$ feet per minute or equivalently $\frac{120\pi}{60} = 2\pi$ ft/sec. $\square$

(c) How fast is Klaus travelling (in feet per second) if he sits at the center of the merry-go-round?

Solution: Since Klaus is at the center his distance from the center is 0 and therefore isn't going anywhere. While his angular velocity is $\frac{2\pi}{5}$ rad per second, his speed is 0. $\square$

3. The functions $\sin x$ and $\cos x$ are almost identical except for horizontal position (see below). That means you should be able to express $\sin x$ as a shift of $\cos x$ and similarly, $\cos x$ as a shift of $\sin x$. Specifically, find c so that $\sin x = \cos(x + c)$ and find c so that $\cos x = \sin(x + c)$.



Solution: In both cases we note that cosine reaches its maximum at $x = 0$, that is$\cos(0) = 1$. Meanwhile sine doesn't reach its maximum until $\frac{\pi}{2}$ radians later. That means in order to get cosine shifted ahead to sine we have to shift it forward by $\frac{\pi}{2}$ radians. Therefore, $\sin x = \cos\left(x - \frac{\pi}{2}\right)$.

Similarly, for sine to catch up with cosine, it has to be shifted ahead by $\frac{\pi}{2}$ radians so $\cos x = \sin\left(x + \frac{\pi}{2}\right)$. $\square$

4. Solve the following for $x \in \mathbb{R}$.

(a) $\sin x = 1$

Solution: On the interval $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ (this is the interval we restrict sine to in order to make arcsin a function) we know $\sin x = 1$ when $x = \arcsin(1) = \frac{\pi}{2}$. Since this is the only occurrence on that interval and sine has a period of 2π, the general solution is $x = \frac{\pi}{2} + 2\pi n$, where $n \in \mathbb{Z}$. $\square$

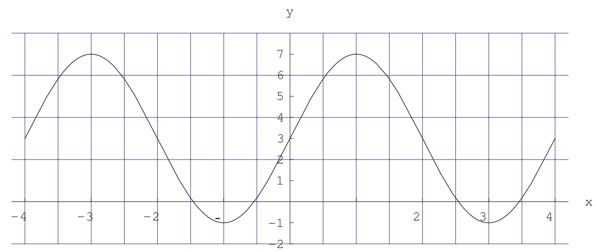
(b) $\cos 3x = 1$

Solution: Similar logic here, cosine is usually restricted to the interval $[0, \pi]$ in order to invert it. Then $3x = \arccos(1) \longrightarrow x = 0$. Since $\cos(3x)$ has a period of $\frac{2\pi}{3}$, the general solution is $x = 0 + \frac{2\pi}{3}n$ $n \in \mathbb{Z}$. $\square$

(c) $\sin(x^2 - 1) = 1$

Solution: From part (a) we know the solution to $\sin(\ ) = 1$ is $x = \frac{\pi}{2} + 2\pi n, n \in \mathbb{Z}$. In this case x is replaced with $x^2 - 1$ so we have $x^2 - 1 = \frac{\pi}{2} + 2\pi n$ $(n \in \mathbb{Z})$. Solving gives us $x = \pm\sqrt{1 + \frac{\pi}{2} + 2\pi n}$, $(n \in \mathbb{Z}_{\geq 0})$. Notice the change from $n \in \mathbb{Z}$ to n in the non-negative integers since the square root has a restricted domain. $\square$

5. Find two different equations for the periodic function shown on the right. (One in terms of $\sin x$ and the other in terms of $\cos x$).



Solution: We need to find the constants A, B, C and D in the form $y = A\sin(Bx + D) + C$

$C = \frac{-1+7}{2} = 3$

$A = 7 - 3 = 4$

The span from peak to peak is 4π (or the span from top to bottom is 2π) so the period is 4π and therefore $B = \frac{1}{2}$.

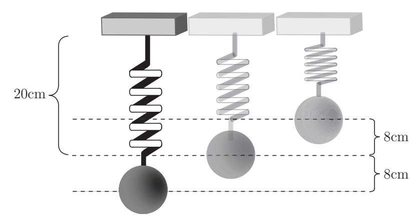
Finally, since the maximum occurs at π when for sine it normally occurs at $\frac{\pi}{2}$, we need to find C so that $\frac{1}{2}(\pi) + D = \pi \longrightarrow D = 0$.

This gives us $y = 4\sin\left(\frac{1}{2}x\right) + 3$ $\square$

Similarly, for cosine we have the same A, B and C so we only need to find D. The maximum or cosine occurs at 0 so we need D so that $\frac{1}{2}(\pi) + D = 0 \longrightarrow D = -\frac{\pi}{2}$.

This gives us $y = 4\cos\left(\frac{1}{2}x - \frac{\pi}{2}\right) + 3$ $\square$

6. A mass is suspended at the end of a spring where it hangs 20cm from the ceiling. It is displaced 8cm below its rest position and released. It reaches the point closest to the ceiling (12cm) after 1 second. Write a periodic model for this situation giving the distance of the mass from the ceiling, y, as a function of time, t.



Solution: Again, we need to find the constants A, B, C and D in the form $y = A\sin(Bx + D) + C$. The amplitude, A is the displacement from center so $A = 8$ if we make the center $C = 20$.

The period is 2 seconds (to return to start) and therefore $B = \pi$. Finally, since the pendulum begins at its maximum displacement we can use cosine (or $-\cos$) and therefore $D = 0$. This gives us $y = 8\cos(\pi t) + 20$. $\square$

7. The table below shows the US average unemployment rate at the beginning of each year from 1993 – 2003. (Where 1993 is $t = 0$).

t (years)	0	1	2	3	4	5	6	7	8	9	10
$U(t)$ (% unemployment)	7.3	6.6	5.6	5.6	5.3	4.6	4.3	4.0	4.1	5.6	5.7

a) Write a periodic function that models these data using methods discussed in class.

Solution: The maximum unemployment is 7.3% while the minimum is 4.0% so $C = \frac{7.3+4}{2} = 5.65$.

Then $A = 7.3 - 5.65 = 1.65$

It takes seven years to go from the highest to the lowest value so 7 years is half a period and therefore the period is 14 years. It follows that $B = \frac{2\pi}{14} = \frac{\pi}{7}$.

Finally, if we use cosine which begins at its maximum like the date set, we can ignore D.

This gives us $U(t) = 1.65\cos\left(\frac{\pi}{7}t\right) + 5.65$ $\square$

b) Use your model to predict the unemployment rate for January of 2016.

Solution: From the equation in (a) we have $U(13) = 1.65\cos\left(\frac{\pi}{7}(23)\right) + 5.65 \approx 4.6\%$ unemployment. $\square$

(Note that your calculator needs to be in radians, not degrees).

8. Answers: (a) 0.5mi (b) 0.8mi (c) 20.2 mph (d) 2min, 19sec.