The derivative of $f(x) = \dfrac{1}{x}$ at $x = 3$:

$$
\begin{aligned}
f'(3) &= \lim_{h\to 0} \frac{\frac{1}{3+h} - \frac{1}{3}}{h} \\
&= \lim_{h\to 0} \frac{\frac{1}{3+h} - \frac{1}{3}}{h} \cdot \frac{3(3+h)}{3(3+h)} \\
&= \lim_{h\to 0} \frac{3-(3+h)}{3h(3+h)} \\
&= \lim_{h\to 0} \frac{-h}{3h(3+h)} \\
&= \lim_{h\to 0} \frac{\cancel{h}^{-1}}{3\cancel{h}^{1}(3+h)} \\
&= \lim_{h\to 0} \frac{-1}{3(3+h)} \\
&= \frac{-1}{9}
\end{aligned}
$$

□