Math 251

Functions and Other Problems Name:____________________

1. The conversion formula relating $°C$ and $°F$ is given by $F = \frac{9}{5}C + 32$. This is often derived from two of the more common reference temperatures: the freezing point of water: $(0°C, 32°F)$ and the boiling point of water: $(100°C, 212°F)$. Show this derivation.

2. Suggest a rule of thumb for converting $°C$ to $°F$: ____________________

   For what temperatures in $°C$ is your approximation . . .

   (a) an underestimate

   (b) an overestimate

   (c) exactly right

   (d) Within $1F°$ of correct

3. Write a piecewise function for the graph shown to the right.

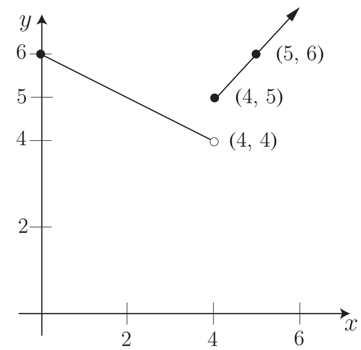


Figure 1: Piecewise Function

4. DRY OXTAIL IN REAR is an anagram for what word?

5. Solve $x^4 - 13x^2 + 36 = 0$

6. Solve $t^{-1/3} - 5t^{2/3} = 0$

7. Solve $\dfrac{x}{\sqrt{1+x^2}} - \dfrac{4-x}{\sqrt{16+(4-x)^2}} = 0$

8. The frustum of a pyramid is the bottom portion of the solid with the top sliced off parallel to the base. The formula for the volume of the frustum of a pyramid is given by $V = \dfrac{a^2+ab+b^2}{3}h$ where a is the side of one of the square bases, b is the side of the other square base, and h is the altitude of the frustum.

   (a) Derive this formula.

   (b) Show that this formula becomes the formula for the volume of a pyramid when $b = 0$.

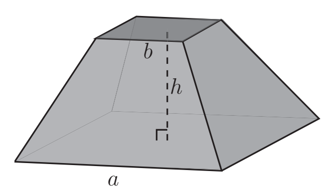


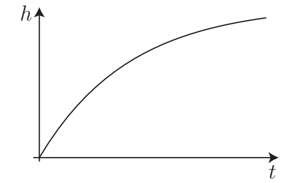
Figure 2: Frustum

9. If $f(x) = ax^2 + bx + c$ determine the largest domain for which f is invertible and find $f^{-1}(x)$.

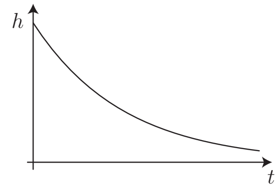
10. San Francisco is located at latitude 37° North. To an observer located in space, how fast (in km/hr) would residents of SF appear to be moving as they rotated around Earth's axis? Assume $r_E \approx 6400$ km.

(11) For each of the curves below indicate whether it is increasing or decreasing and whether it is concave up or down.

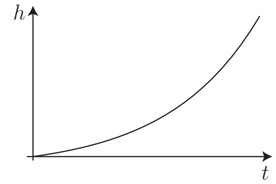
(a)



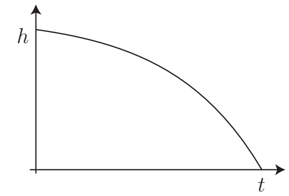
(b)



(c)

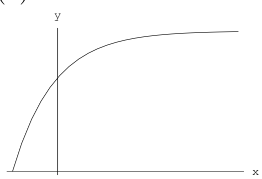


(d)



(12) Write a possible equation for each curve graphed below.

(a)



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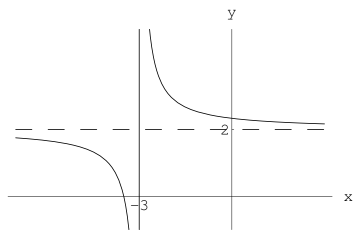

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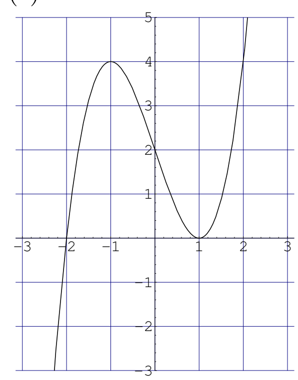
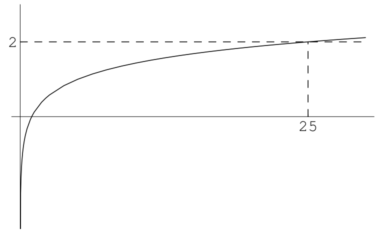
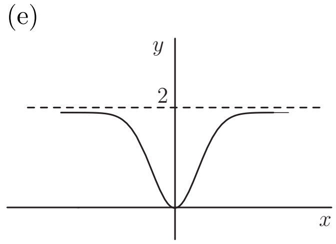


(d)



(e)



(13) The function $g(x) = \dfrac{\sin x}{x}$ is defined for all real $x \neq 0$. Define a function $f(x)$ that is identical to g for $x \neq 0$ but is defined and continuous for all real numbers.