Chp 6 Problems Name:

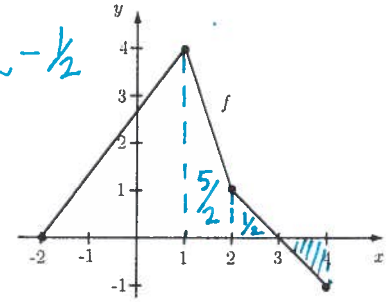
Show all relevant work!

1. The graph of the function f, consisting of three line segments, is given below.
Let $g(x) = \int_1^x f(t)\,dt$. Compute (and show your reasoning) . . .

(a) $g(4)$: $5/2$ $g(4) = \int_1^4 f(t)\,dt = 5/2 + 1/2 - 1/2$

(b) $g(-2)$: -6 $g(-2) = \int_1^{-2} f(t)\,dt = -\int_{-2}^{1} f(t)\,dt$

$-\frac{1}{2}(3)(4)$



2. Use calculus to help you write the equation of the line tangent to $f(x) = \int_1^{x^3} e^{1-t^2}\,dt$ at $x = 1$.
Use it to approximate the value of $f(1.4)$.

SLOPE: $f'(1) = 3x^2 e^{1-x^6} = 3$

POINT: $f(1) = \int_1^1 e^{1-t^2}\,dt = 0 \Rightarrow (1,0)$

$\boxed{y = 3x - 3}$

$f(1.4) \approx 3(1.4) - 3 = 2.2$.

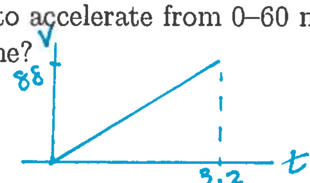
3. Where is the function, $f(x)$, in question #3 at its minimum?

$f'(x) = 3x^2 e^{1-x^6} \rightarrow 0$ AT $x = 0$ BUT f' DOESN'T CHANGE SIGNS SO $f' \geq 0$ $\forall x$ AND THEREFORE f INCREASING. f MINIMUM AT LEFT E.P. OF ANY INTER.

4. Suppose $\int_0^1 f(x)\,dx = k$, where k is a constant. Evaluate $\int_0^1 x f(1-x^2)\,dx$ and give your answer in terms of k.

$u = 1 - x^2$
$du = -2x\,dx \rightarrow -\frac{1}{2}du = x\,dx \Rightarrow -\frac{1}{2}\int_1^0 f(u)\,du = \frac{1}{2}\int_0^1 f(u)\,du$
$u(0) = 1;\ u(1) = 0$
$= \boxed{\frac{1}{2}k}$

5. The Tesla Model S P85D is reported to accelerate from 0–60 mph (88 ft/sec) in 3.2 seconds.
How far does it travel during that time?



CHANGE IN POSITION IS $\int_0^{3.2} v\,dt = \frac{1}{2}(3.2)(88)$
$= 140.8$ ft.

6. With t in years since 2000, the population, P, of the world in billions can be modeled by $P = 6.1e^{0.012t}$.
Use the Fundamental Theorem to predict the average population of the world between 2000 and 2010.

$\frac{1}{10}\int_0^{10} 6.1e^{0.012t}\,dt$ $u = 0.012t$, $du = 0.012\,dt$, $\frac{1}{0.012}du = dt$ $\frac{6.1}{(10)(0.012)}\int_0^{0.12} e^u\,du = \frac{6.1}{0.12}(e^{0.12} - 1)$

≈ 6.48 BILLION

7. My dog likes to walk according to the function $v(t) = t\sin t$ where t is in seconds and $v(t)$ is her velocity in feet per second as she walks north (+) and south (–), relative to her house.

(a) Find the formula (using calculus) for her position, $s(t)$, relative to home as a function of time. Assume that when she begins walking she is 20 feet north of her house.

$S(t) = \int_0^t x\sin x\,dx$ (SEE BELOW) $= \sin t - t\cos t + 20$

(b) She reverses direction often after she starts walking. When are the first three times she reverses direction? $V(t) = t\sin t = 0$ FOR $t = \pi, 2\pi, 3\pi$.

THIS IS WHERE $V(t)$ CHANGES SIGN AND DOG CHANGES DIR.

(c) Assume she stops walking after $t = 3\pi$ seconds. Find her average velocity since she started walking.

$\frac{1}{3\pi - 0}\int_0^{3\pi} t\sin t\,dt = \frac{1}{3\pi}(\sin t - t\cos t)\Big|_0^{3\pi}$
$= \frac{1}{3\pi}(3\pi) = 1$ ft/sec.

(d) Repeat (c) for her average *speed* over this time.

$\frac{1}{3\pi}\int_0^{3\pi} |t\sin t|\,dt = \frac{1}{3\pi}\left[\int_0^{\pi} V(t)\,dt - \int_{\pi}^{2\pi} V(t)\,dt + \int_{2\pi}^{3\pi} V(t)\,dt\right]$

$= \frac{1}{3\pi}(\pi + 3\pi + 5\pi) = 3$ ft/sec

$\boxed{7.}$ (a) $\displaystyle\int_0^t x \sin x \, dx$

$u = x \qquad v' = \sin x$

$du = dx \qquad v = -\cos x$

$$\int x \sin x \, dx = -x\cos x + \int \cos x \, dx$$

$$= -x\cos x + \sin x + C$$

$$\int_0^t x \sin x \, dx = -x\cos x + \sin x \Big|_0^t = \sin t - t\cos t$$