Math 252

Problems   Name:

YOU MAY USE A CALCULATOR TO VERIFY SOLUTIONS, BUT NOT TO PROVIDE THEM.

Show all relevant work!

1. Find the antiderivatives below using either substitution or integration by parts.

(a) $\int \frac{\ln x}{x}\,dx$ (b) $\int \frac{t}{e^t}\,dt$ (c) $\int \frac{x-1}{\sqrt{x+1}}\,dx$

(a)
$u = \ln x$
$du = \frac{1}{x}dx$
$\int u\,du$
$= \frac{1}{2}u^2 + C$
$= \frac{1}{2}(\ln x)^2 + C$

(b)
$\int te^{-t}dt$
$u = t \quad v' = e^{-t}$
$u' = 1dt \quad v = -e^{-t}$
$-te^{-t} + \int e^{-t}dt$
$= -te^{-t} - e^{-t} + C$
or $-e^{-t}(t+1) + C$

(c)
$u = x+1 \to u-1 = x$
$du = dx$
$\int \frac{u-2}{u^{1/2}}du$
$\int u^{1/2} - 2u^{-1/2}\,du$
$2/3\,u^{3/2} - 4u^{1/2} + C$
$2/3(x+1)^{3/2} - 4\sqrt{x+1} + C$

2. Suppose $\int_{-1}^{0} f(x)\,dx = 2$. Find $\int_0^1 x f(x^2-1)\,dx$.

$u = x^2 - 1$
$du = 2x\,dx$
$\frac{1}{2}du = x\,dx$

$u(1) = 0$, $u(0) = -1$

$\int_0^1 x f(x^2-1)dx \longrightarrow \frac{1}{2}\int_{-1}^{0} f(u)\,du = \frac{1}{2}(2) = 1.$

3. Use the FTC to help you find the RMS value for $f(t) = \cos(120\pi t)$ over one period: $\sqrt{60\int_0^{1/60} \cos^2(120\pi t)\,dt}$

consider $\int \cos^2 x\,dx$

$u = \cos x \quad v' = \cos x$
$u' = -\sin x \quad v = \sin x$

$\int \cos^2 x\,dx = \cos x \sin x + \int \sin^2 x\,dx$
$= \cos x \sin x + \int 1 - \cos^2 x\,dx$
$= \cos x \sin x + \int 1dx - \int \cos^2 x\,dx$
$2\int \cos^2 x\,dx = \cos x \sin x + x$
$\int \cos^2 x\,dx = (\cos x \sin x + x)/2$

$60\int_0^{1/60} \cos^2(120\pi t)\,dt$
$u = 120\pi t \to u(0) = 0$, $u(1/60) = 2\pi$
$\frac{1}{120\pi}du = dt$
$\frac{60}{120\pi}\int_0^{2\pi} \cos^2 u\,du$
$\frac{60}{120\pi}\left.\frac{(\cos u \sin u + u)}{2}\right|_0^{2\pi}$
$= \frac{1}{2\pi}\cdot\frac{2\pi}{2} = 1/2$

So RMS: $\sqrt{1/2} \approx 0.707$.

4. Find the area between $y = \ln x$ and $y = x - 2$.

Find $\ln x = x - 2$ (using calculator)
$(0.16, -1.84)$ & $(3.15, 1.15)$

$\int_{0.16}^{3.15} \ln x - (x-2)\,dx \approx \boxed{1.85}$