Exam 3 Name:________________________

YOU MAY USE A CALCULATOR TO COMPUTE SOLUTIONS BUT SHOW YOUR SET-UPS.

Show all relevant work!

Some useful(?) formulas:

$C = 2\pi r$ $A = \pi r^2$ $SA = 4\pi r^2$ $V = \dfrac{4}{3}\pi r^3$ $\cosh^2 x - \sinh^2 x = 1$

Arclength: $\ell = \displaystyle\int_a^b \sqrt{1 + [f'(x)]^2}\,\mathrm{d}x$ and $\ell = \displaystyle\int_\alpha^\beta \sqrt{\left(\frac{\mathrm{d}x}{\mathrm{d}\theta}\right)^2 + \left(\frac{\mathrm{d}y}{\mathrm{d}\theta}\right)^2}\,\mathrm{d}\theta$ Area $= \dfrac{1}{2}\displaystyle\int_\alpha^\beta [f(\theta)]^2\,\mathrm{d}\theta$ $\overline{x} = \dfrac{\int x\,\mathrm{d}m}{\int \mathrm{d}m}$

DON'T PANIC

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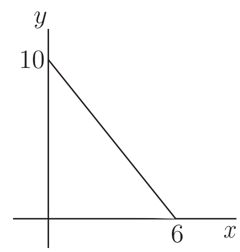
(1) Set up (but do not evaluate) the integrals to find the center of mass of the triangular plate of uniform thickness and density shown below.

$\overline{x} =$____________________, $\overline{y} =$____________________

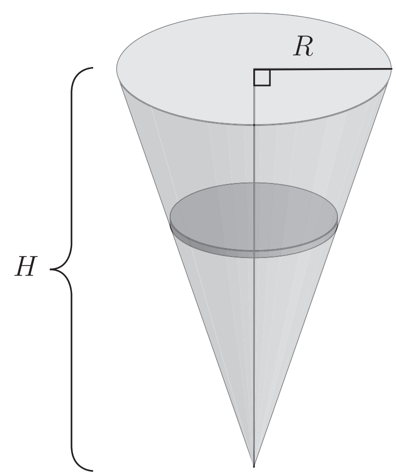


See 8.4 Example 8, Work and Density problems #3, problems 8.4.13, 26-28, class notes for 8.4

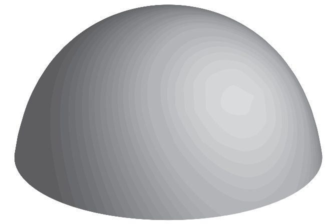
(2) Show that the vertical center of mass for a cone of height H and base radius R with uniform density, δ, lies on the axis of symmetry (altitude) $\frac{3}{4}$ of the distance from the vertex.

See 8.4.29; class notes from 8.1 for volume; Closing Problems H/O #4 and 8.5.22 for examples with work

Note: most of these problems are set up to make things easier – so don't go out of your way to make them harder. Make y (or h) the distance from the vertex - not the distance from the base (top, in this picture).

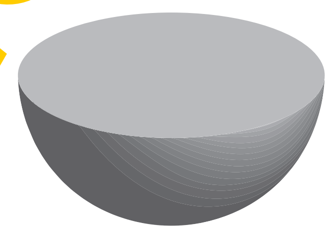


(3) A hemispherical tank of radius 16 feet is filled with oil. Find the work done by gravity in emptying the tank through the bottom. Assume the weight density of the oil is 42 lbs/ft^3.

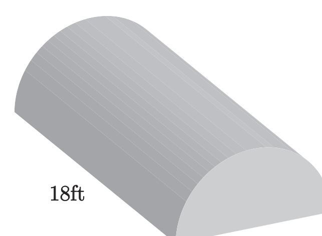


See Closing Problems H/O #4/5, Work Density Problems #5, and 8.5.23 for similar work examples.
See 8.4 Example 9 for similar example of weight

(4) Repeat number ((3)) if the tank is turned upside down (but still drained by gravity down through the bottom).



(5) A storage shed in the shape of a half-cylinder width length 18ft and width 12ft is filled with sawdust. The sawdust at the bottom is more dense than that at the top and we model it with the function $\delta(y) = \dfrac{k}{y+1}$ lb.s/ft^3, where k is a constant associated with the type of sawdust. Write an integral representing the total weight of the sawdust in the shed.



See 8.4.32 (set up)
and 8.5.25 for problem with work.

(6) The following table gives the density D (in 10^{12} kg/(km)^3) of the Earth at a depth x km below the Earth's surface. The radius of the Earth is about 6370 km. Find an upper estimate of the Earth's mass.

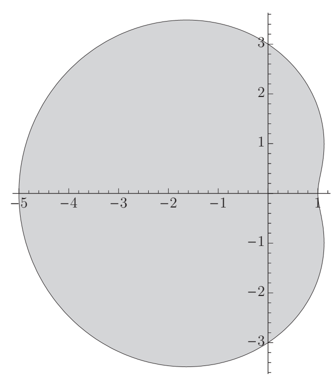
x (km)	0	1000	2000	2900	3000	4000	5000	6000	6370
D ($\times 10^{12}$ kg/(km)^3)	3.3	4.5	5.1	5.6	10.1	11.4	12.6	13.0	13.0

See 8.4.34, 8.4.35, If you use solutions manual, ignore differences in volume and think spherical shells instead.

(7) The population density of a town varies with the radial distance from the town center according to the model $\delta(r) = 10e^{-0.02r}$, measured in thousands of people per square kilometer. If the entire town is confined to a circle 4km in radius, set up an integral that gives the total population of the town.

See 8.4 Example 4 and problems 8.4.16, 8.4.17 and Closing Problems #9

(8) Find the area bounded by the cardioid $r = 3 - 2\cos(\theta)$.



EC Two long, thin, uniform rods of lengths l_1 and l_2 lie on a straight line with a gap between them of length a. Suppose their masses are M_1 and M_2, respectively, and the constant of gravitation is G. What is the force of attraction between the rods.⟨ Hint: consider one rod at a time ...⟩