

# USE INFINITE SERIES TO AMUSE ASTONISH

1-40 ■ Test the series for convergence or divergence.

$$1. \sum_{n=1}^{\infty} \frac{\sqrt{n}}{n^2 + 1}$$

$$3. \sum_{n=1}^{\infty} \frac{4^n}{3^{2n-1}}$$

$$5. \sum_{n=2}^{\infty} \frac{(-1)^n}{(\ln n)^2}$$

$$7. \sum_{k=1}^{\infty} k^{-1.7}$$

$$9. \sum_{n=1}^{\infty} \frac{n}{e^n}$$

$$11. \sum_{n=2}^{\infty} \frac{n^3 + 1}{n^3 - 1}$$

$$13. \sum_{n=2}^{\infty} \frac{2}{n(\ln n)^3}$$

$$15. \sum_{n=1}^{\infty} \frac{3^n n^2}{n!}$$

$$17. \sum_{n=1}^{\infty} \frac{3^n}{5^n + n}$$

$$19. \sum_{n=0}^{\infty} \frac{n!}{2 \cdot 5 \cdot 8 \cdot \dots \cdot (3n + 2)}$$

$$2. \sum_{n=1}^{\infty} \cos n$$

$$4. \sum_{i=1}^{\infty} \frac{i^4}{4^i}$$

$$6. \sum_{n=1}^{\infty} n^2 e^{-n^3}$$

$$8. \sum_{n=0}^{\infty} \frac{10^n}{n!}$$

$$10. \sum_{m=1}^{\infty} \frac{2m}{8m - 5}$$

$$12. \sum_{n=1}^{\infty} \left( \frac{n^2 + 1}{2n^2 + 1} \right)^n$$

$$14. \sum_{n=1}^{\infty} \frac{\sqrt{n}}{e^{\sqrt{n}}}$$

$$16. \sum_{n=1}^{\infty} \frac{3}{4n - 5}$$

$$18. \sum_{k=1}^{\infty} \frac{k + 5}{5^k}$$

$$20. \sum_{n=1}^{\infty} \frac{(-1)^n n}{(n + 1)(n + 2)}$$

$$21. \sum_{i=1}^{\infty} \frac{1}{\sqrt{i(i + 1)}}$$

$$23. \sum_{n=1}^{\infty} (-1)^n 2^{1/n}$$

$$25. \sum_{n=1}^{\infty} (-1)^n \frac{\ln n}{\sqrt{n}}$$

$$27. \sum_{n=0}^{\infty} (-\pi)^n$$

$$29. \sum_{n=1}^{\infty} \frac{(-2)^{2n}}{n^n}$$

$$31. \sum_{k=1}^{\infty} \frac{k \ln k}{(k + 1)^3}$$

$$33. \sum_{n=1}^{\infty} \frac{2^n}{(2n + 1)!}$$

$$35. \sum_{n=1}^{\infty} \frac{\tan^{-1} n}{n \sqrt{n}}$$

$$37. \sum_{n=1}^{\infty} \left( \frac{n}{n + 1} \right)^{n^2}$$

$$39. \sum_{n=1}^{\infty} (\sqrt[3]{2} - 1)^n$$

$$22. \sum_{n=1}^{\infty} \frac{n^2}{\sqrt{n^5 + n^2 + 2}}$$

$$24. \sum_{n=1}^{\infty} \frac{\cos(n/2)}{n^2 + 4n}$$

$$26. \sum_{n=1}^{\infty} \frac{\tan(1/n)}{n}$$

$$28. \sum_{n=1}^{\infty} \frac{\sqrt[3]{n} + 1}{n(\sqrt{n} + 1)}$$

$$30. \sum_{n=1}^{\infty} \frac{2^{3n-1}}{n^2 + 1}$$

$$32. \sum_{n=1}^{\infty} \frac{e^{1/n}}{n^2}$$

$$34. \sum_{j=1}^{\infty} (-1)^j \frac{\sqrt{j}}{j + 5}$$

$$36. \sum_{n=1}^{\infty} \frac{(2n)^n}{n^{2n}}$$

$$38. \sum_{n=2}^{\infty} \frac{1}{(\ln n)^{\ln n}}$$

$$40. \sum_{n=1}^{\infty} (\sqrt[3]{2} - 1)^n$$

1-32 ■ Determine whether the series is absolutely convergent, conditionally convergent, or divergent.

$$1. \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n \sqrt{n}}$$

$$2. \sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}}$$

$$3. \sum_{n=1}^{\infty} \frac{(-3)^n}{n^3}$$

$$4. \sum_{n=0}^{\infty} \frac{(-3)^n}{n!}$$

$$5. \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{2n + 1}$$

$$6. \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n^2 + 1}$$

$$7. \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{(2n - 1)!}$$

$$8. \sum_{n=1}^{\infty} e^{-n} n!$$

$$9. \sum_{n=1}^{\infty} (-1)^n \frac{n}{n^2 + 4}$$

$$10. \sum_{n=1}^{\infty} (-1)^{n-1} \frac{\sqrt{n}}{n + 1}$$

$$11. \sum_{n=1}^{\infty} (-1)^n \frac{2n}{3n - 4}$$

$$12. \sum_{n=1}^{\infty} (-1)^n \frac{2^n}{n^2 + 1}$$

$$13. \sum_{n=1}^{\infty} \frac{\sin 2n}{n^2}$$

$$15. \sum_{n=1}^{\infty} \frac{(-2)^n}{n 3^{n+1}}$$

$$17. \sum_{n=1}^{\infty} \frac{(n + 1)5^n}{n 3^{2n}}$$

$$19. \sum_{n=1}^{\infty} \frac{n!}{(-10)^n}$$

$$21. \sum_{n=1}^{\infty} \frac{\cos(n\pi/3)}{n!}$$

$$23. \sum_{n=1}^{\infty} \frac{(-n)^n}{5^{2n+3}}$$

$$25. \sum_{n=1}^{\infty} \left( \frac{1 - 3n}{3 + 4n} \right)^n$$

$$14. \sum_{n=1}^{\infty} \frac{(-1)^n \arctan n}{n^3}$$

$$16. \sum_{n=1}^{\infty} \frac{(-1)^{n-1} 5^{n-1}}{(n + 1)^2 4^{n-2}}$$

$$18. \sum_{n=1}^{\infty} \frac{\cos(n\pi/6)}{n \sqrt{n}}$$

$$20. \sum_{n=1}^{\infty} \frac{n!}{n^n}$$

$$22. \sum_{n=2}^{\infty} \frac{(-1)^n}{(\ln n)^n}$$

$$24. \sum_{n=2}^{\infty} \frac{(-1)^n}{n \ln n}$$

$$26. \sum_{n=1}^{\infty} \frac{(-2)^n n^2}{(n + 2)!}$$

# ANSWERS

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|--|---|
| 1. Conv. (Comp. $\sqrt[n]{1/n^{3/2}}$ )              | 21. Div (lim Comp $\sqrt[n]{1/i}$ )                   |
| 2. Div. ( $\lim_{n \rightarrow \infty} a_n \neq 0$ ) | 22. Conv. (Comp of $\sqrt[n]{1/n^{3/2}}$ )            |
| 3. Conv. (Comp. $\sqrt[n]{6/5} (\frac{6}{5})^n$ )    | 23. Div ( $\lim_{n \rightarrow \infty} a_n \neq 0$ )  |
| 4. Conv. (RATIO)                                     | 24. Conv. (Comp. $\sqrt[n]{1/n^2}$ )                  |
| 5. Conv. (A/S)                                       | 25. Conv (A/S)  |
| 6. Conv. (INT.)                                      | 26. Div (Comp of $\sqrt[n]{1/n}$ )                    |
| 7. Conv. (P-test, $p > 1$ )                          | 27. Div ( $\lim_{n \rightarrow \infty} a_n \neq 0$ )  |
| 8. Conv. (RATIO)                                     | 28. Conv. (Comp $\sqrt[n]{1/n^{7/6}}$ )               |
| 9. Conv. (INT. - BY PARTS)                           | 29. (Conv. (ROOT))                                    |
| 10. Div. (Comp $\sqrt[n]{1/n}$ )                     | 30. Div ( $\lim_{n \rightarrow \infty} a_n \neq 0$ )  |
| 11. Div. (Comp $\sqrt[n]{1/n}$ )                     | 31. Conv. (Comp $\sqrt[n]{1/k^{3/2}}$ )               |
| 12. (Div - (ROOT))                                   | 32. Conv. (lim Comp or Comp $\sqrt[n]{2/n^2}$ )       |
| 13. Conv. (INT.)                                     | 33. Conv. (RATIO)                                     |
| 14. Conv. (INT.)                                     | 34. Div. ( $\lim_{n \rightarrow \infty} a_n \neq 0$ ) |
| 15. Conv. (RATIO)                                    | 35. Conv. (Comp $\sqrt[n]{4/n^{3/2}}$ )               |
| 16. Div. (Comp. $\sqrt[n]{4/n}$ )                    | 36. (Div. (ROOT))                                     |
| 17. Conv. (Comp. $\sqrt[n]{6/5} (\frac{6}{5})^n$ )   | 37. Div ( $\lim_{n \rightarrow \infty} a_n \neq 0$ )  |
| 18. Conv. (Comp $\sqrt[n]{6/5} (\frac{6}{5})^n$ )    | 38. (Conv. (ROOT))                                    |
| 19. Conv. (RATIO)                                    | 39. (Conv. ROOT)                                      |
| 20. Conv. (A/S)                                      |   |