12.4 Notes

Linear Functions in $\mathbb{R}^3$

(1.) Sketch the graph (trace) of $6x + 5y - 3z = 30$.

12.4 Notes

Linear Functions in $\mathbb{R}^3$

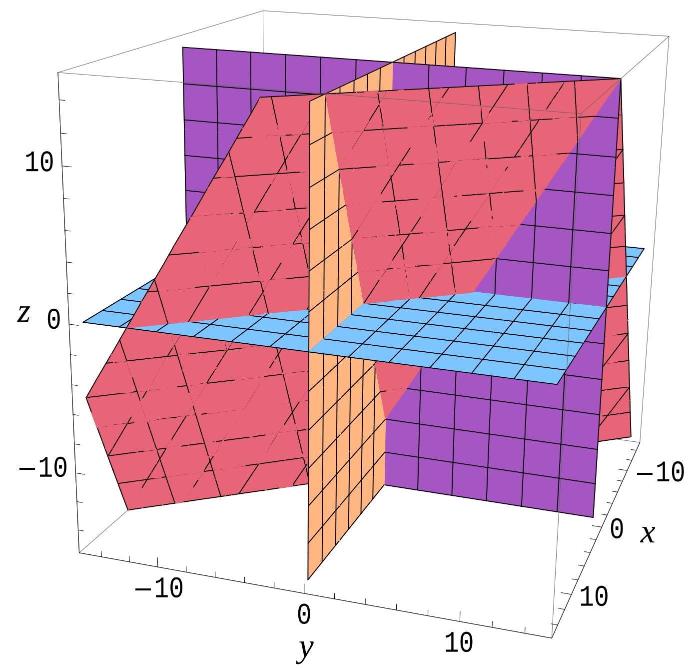
(1.) Sketch the graph (trace) of $6x + 5y - 3z = 30$.

x	y	z
0	0	____
0	____	0
____	0	0

Linear Functions in $\mathbb{R}^3$

(1.) Sketch the graph (trace) of $6x + 5y - 3z = 30$.

x	y	z
0	0	-10
0	6	0
5	0	0



Note, if we solve $6x + 5y - 3z = 30$ for z we have

$$z = f(x, y) = 2x + \frac{5}{3}y - 10$$

Note, if we solve $6x + 5y - 3z = 30$ for z we have

$$z = f(x, y) = 2x + \frac{5}{3}y - 10$$

In the xz–plane we have a slope of $\dfrac{\Delta z}{\Delta x} = \dfrac{2}{1}$ and in the yz–plane we have a slope of $\dfrac{\Delta z}{\Delta y} = \dfrac{5}{3}$.

Example: Find an equation for the linear function with the table below.

$x \backslash y$	10	20	30	40
100	3	6	9	12
200	2	5	8	11
300	1	4	7	10
400	0	3	6	9

Example: Find an equation for the plane containing $(1, 2, 3)$, $(4, -1, -2)$, and $(-3, 1, 1)$.

Example: Find an equation for the plane containing $(1, 2, 3)$, $(4, -1, -2)$, and $(-3, 1, 1)$.

Solution:

We want a solution of the form $z = mx + ny + z_0$ so substituting into the equation gives us:

$$\begin{aligned} 3 &= m + 2n + z_0 && (1) \\ -2 &= 4m - n + z_0 && (2) \\ 1 &= -3m + n + z_0 && (3) \end{aligned}$$

The difference of (1) and (2) gives $5 = -3m + 3n$ and the difference of (2) and (3) gives $-3 = 7m - 2n$ this produces:

$$\begin{aligned} 5 &= -3m + 3n && (4) \\ -3 &= 7m - 2n && (5) \end{aligned}$$

Multiplying (4) by 2 and (5) by 3 gives

$$\begin{aligned} 10 &= -6m + 6n && (6) \\ -9 &= 21m - 6n && (7) \end{aligned}$$

Solving gives us $m = \dfrac{1}{15}$

Since $m = \frac{1}{15}$ it follows

$$5 = -3\left(\frac{1}{15}\right) + 3n \longrightarrow n = \frac{26}{15}$$

And $3 = \frac{1}{15} + 2\left(\frac{26}{15}\right) + z_0 \longrightarrow z_0 = -\frac{8}{15}$

Then we have $z = \frac{1}{15}x + \frac{26}{15}y - \frac{8}{15}$