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REVIEW EXERCISES AND PROBLEMS FOR CHAPTER 12

EXERCISES

1.

Which of the points $A = (23, 92, 48)$, $B = (-60, 0, 0)$, $C = (60, 1, -92)$ is closest to the yz-plane? Which lies on the xz-plane? Which is farthest from the xy-plane?

ANSWER

WORKED SOLUTION

2.

You are at the point $(-1, -3, -3)$, standing upright and facing the yz-plane. You walk 2 units forward, turn left, and walk for another 2 units. What is your final position? From the point of view of an observer looking at the coordinate system in Figure 12.2, are you in front of or behind the yz-plane? To the left or to the right of the xz-plane? Above or below the xy-plane?

3.

On a set of x, y, and z axes oriented as in Figure 12.5, draw a straight line through the origin, lying in the xz-plane and such that if you move along the line with your x-coordinate increasing, your z-coordinate is decreasing.

ANSWER

In Exercises 4-6, determine if z is a function of x and y. If so, find a formula for the function.

4.

$6x - 4y + 2z = 10$

5.

$x^2 + y^2 + z^2 = 100$

ANSWER

WORKED SOLUTION

6.

$3x^2 - 5y^2 + 5z = 10 + x + y$

7.

Figure 12.103 shows the parabolas $z = f(x, b)$ for $b = -2, -1, 0, 1, 2$. Which of the graphs of $z = f(x, y)$ in Figure 12.104 best fits this information?

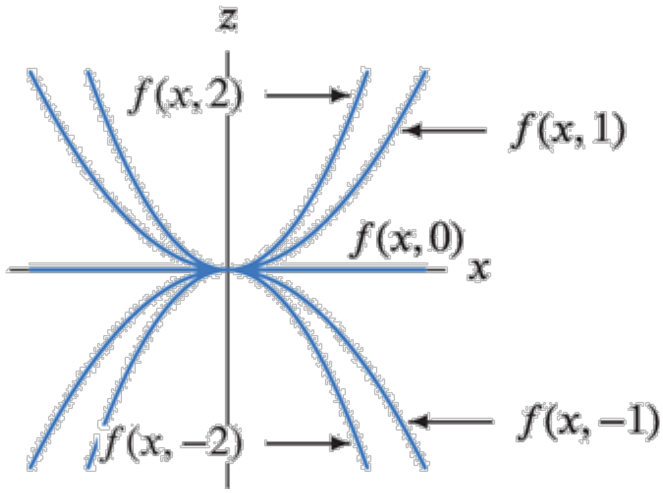


Figure 12.103

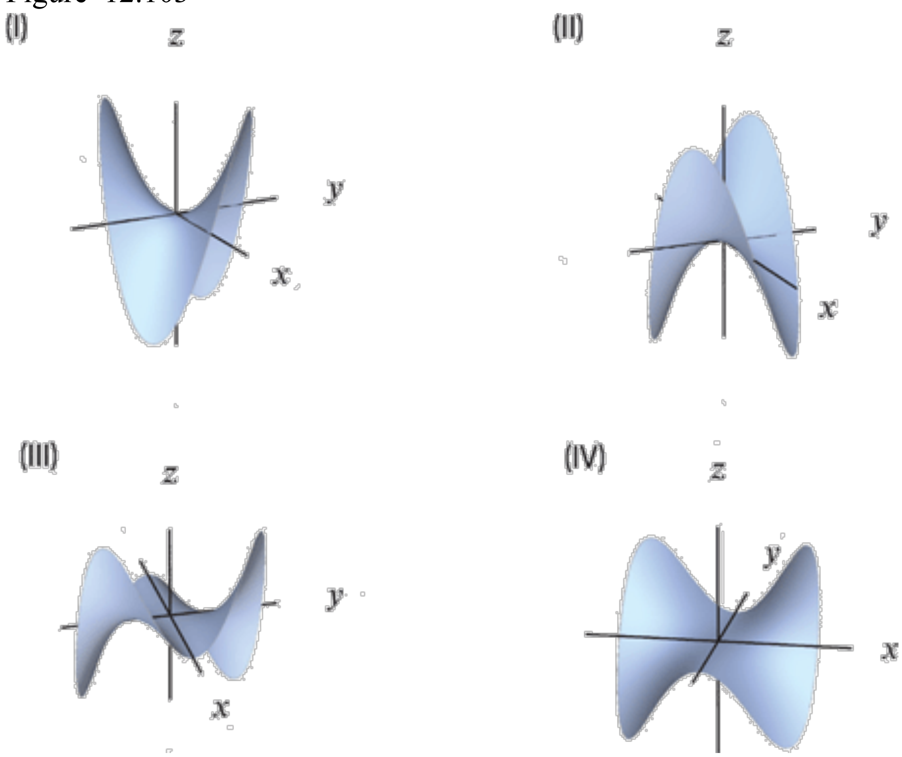


Figure 12.104

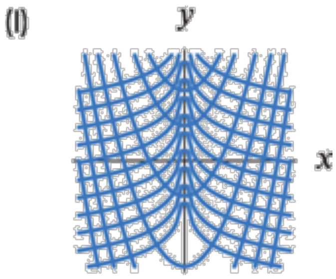
ANSWER

8.

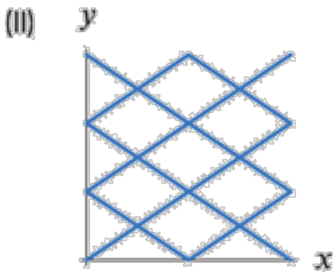
Match the pairs of functions (a)-(d) with the contour diagrams (I)-(IV). In each case, which contours represent f and which represent g? (The x- and y-scales are equal.)

- (a) $f(x, y) = x + y$, $g(x, y) = x - y$
- (b) $f(x, y) = 2x + 3y$, $g(x, y) = 2x - 3y$
- (c) $f(x, y) = x^2 - y$, $g(x, y) = 2y + \ln |x|$
- (d) $f(x, y) = x^2 - y^2$, $g(x, y) = xy$

- I.



- II.



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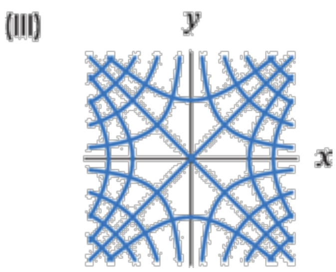

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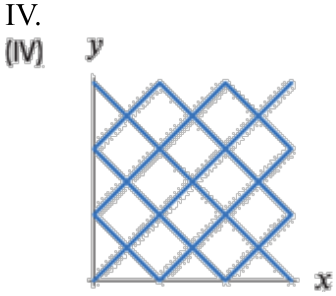
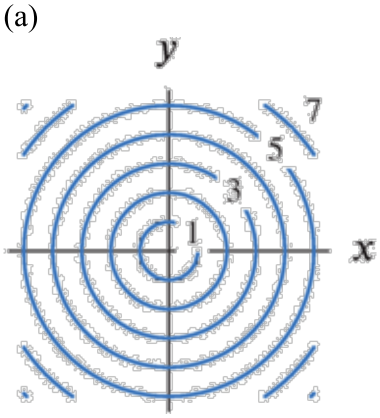


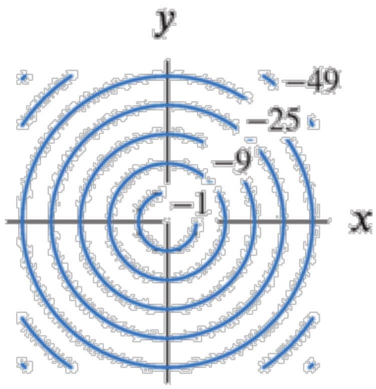
9.

Match the contour diagrams (a)-(d) with the surfaces (I)-(IV). Give reasons for your choice.

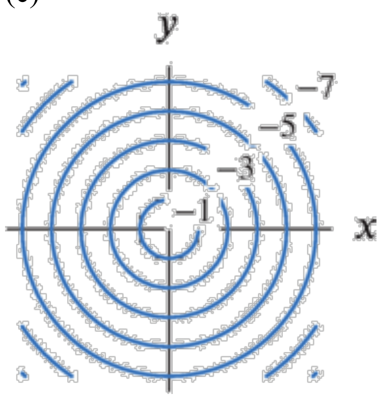
- (a)



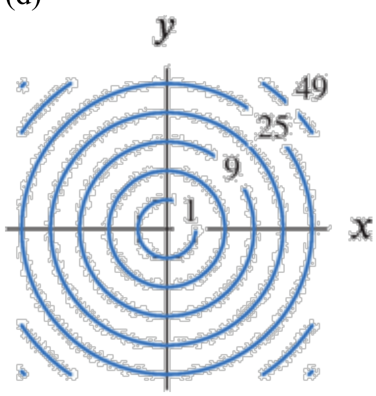
- (b)



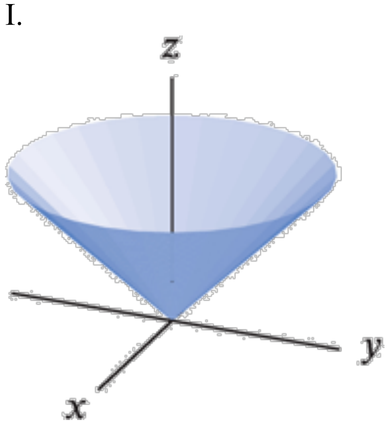
- (c)



- (d)



- I.



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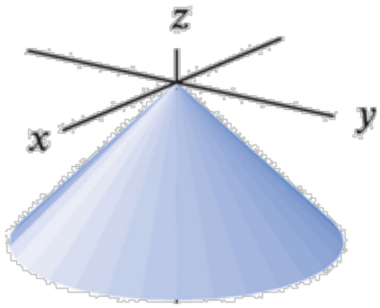
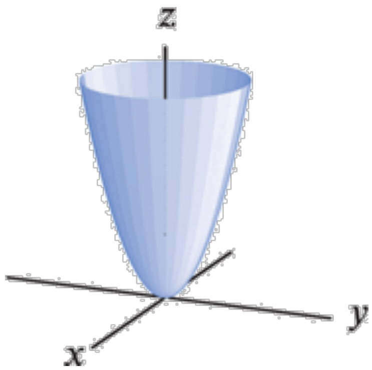
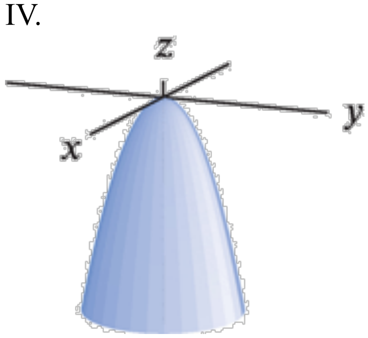


- III.



- IV.



ANSWER

WORKED SOLUTION

In Exercises 10-13, make a contour plot for the function in the region $-2 < x < 2$ and $-2 < y < 2$. What is the equation and the shape of the contours?

10.

$z = 3x - 5y + 1$

11.

$z = \sin y$

ANSWER

12.

$z = 2x^2 + y^2$

13.

$$z = e^{-2x^2 - y^2}$$

ANSWER ⊕

WORKED SOLUTION ⊕

14.

Describe the set of points whose x coordinate is 2 and whose y coordinate is 1.

15.

Find the equation of the sphere of radius 5 centered at (1, 2, 3).

ANSWER ⊕

16.

Find the equation of the plane through the points (0, 0, 2), (0, 3, 0), (5, 0, 0).

17.

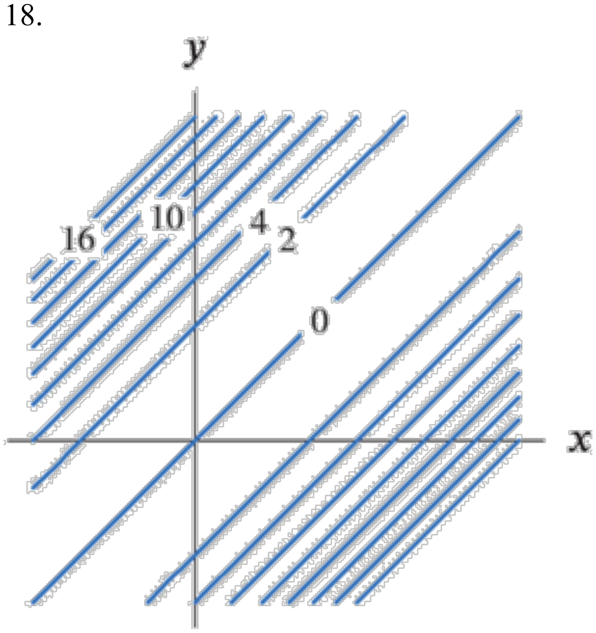
Find the center and radius of the sphere with equation $x^2 + 4x + y^2 - 6y + z^2 + 12z = 0$.

ANSWER ⊕

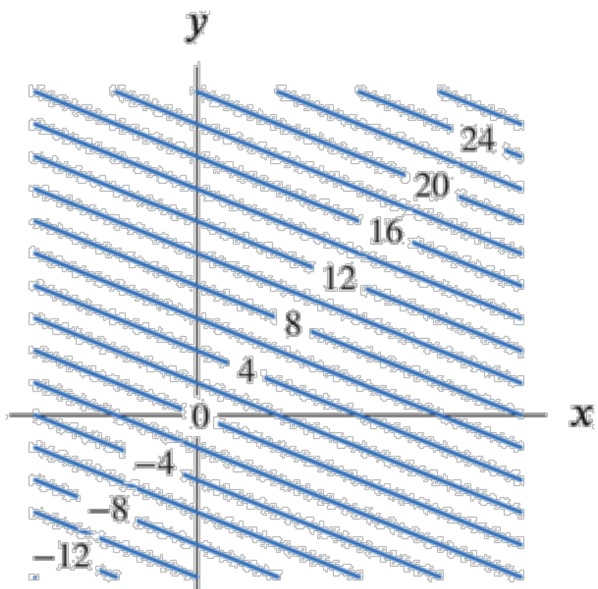
WORKED SOLUTION ⊕

Which of the contour diagrams in Exercises 18-19 could represent linear functions?

18.



19.



ANSWER

20.

(a)

Complete the table with values of a linear function $f(x, y)$.

y

2.53.03.50

−1 6 8

x 1 1 2

3 −6

(b)

Find a formula for $f(x, y)$.

21.

Find a formula for a function $f(x, y, z)$ whose level surfaces look like those in Figure 12.105.

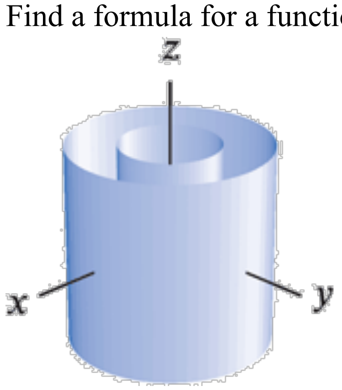


Figure 12.105

ANSWER

WORKED SOLUTION

In Exercises 22-25, represent the surface as the graph of a function, $f(x, y)$, and by level surfaces of the form $g(x, y, z) = c$. (There are many possible answers.)

22.
Paraboloid obtained by shifting $z = x^2 + y^2$ vertically 5 units

23.
Plane with intercepts $x = 2$, $y = 3$, $z = 4$.
ANSWER

24.
Upper half of unit sphere centered at the origin.

25.
Lower half of sphere of radius 2 centered at (3, 0, 0).
ANSWER
WORKED SOLUTION

26.
Describe in words the level surfaces of the function $g(x, y, z) = \cos(x + y + z)$.

Use the catalog to identify the surfaces in Exercises 27-28.
27.
$x^2 + z^2 = 1$
ANSWER

28.
$-x^2 + y^2 - z^2 = 0$

29.
- (a)What features of the contour diagram of $g(x, y)$ in Figure 12.106 suggest that g is linear?

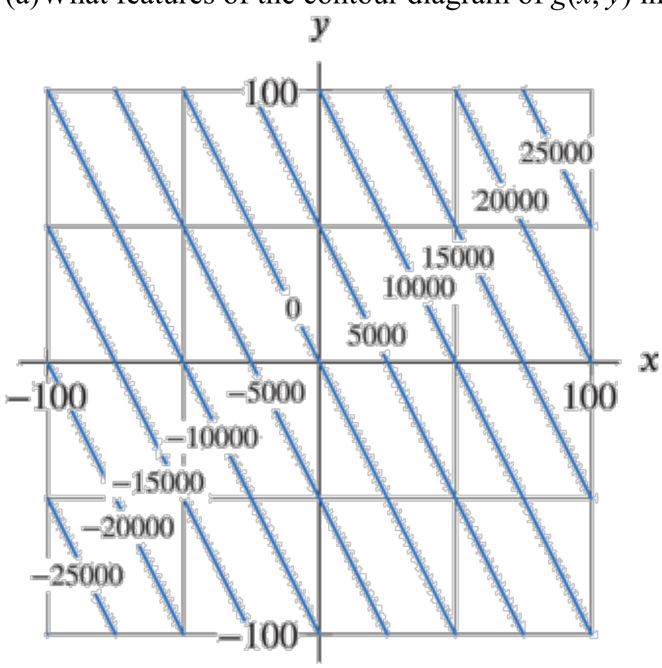


Figure 12.106

- (b)Assuming g is linear, find a formula for $g(x, y)$.

ANSWER

WORKED SOLUTION

PROBLEMS

30.

Use a computer or calculator to draw the graph of the vibrating guitar string function:

$$g(x, t) = \cos\ t\ \sin\ 2x, \quad 0 \le x \le \pi, \quad 0 \le t \le 2\pi.$$

()

Relate the shape of the graph to the cross-sections with t fixed and those with x fixed.

31.

Consider the Cobb-Douglas production function $P = f(L, K) = 1.01L^{0.75}K^{0.25}$. What is the effect on production of doubling both labor and capital?

ANSWER

32.

(a)

Sketch level curves of $f(x, y) = \sqrt{x^2 + y^2} + x$ for $f = 1, 2, 3$.

(b)

For what values of c can level curves $f = c$ be drawn?

33.

Values of $f\ (x, y) = \frac{1}{2}(x + y - 2)\,(x + y - 1) + y$ are in Table 12.13.

Table12.13

		y					
		1	2	3	4	5	6
	1	1	3	6	10	15	21
	2	2	5	9	14	20	
	3	4	8	13	19		
x	4	7	12	18			
	5	11	17				
	6	16					

(a)

Find a pattern in the table. Make a conjecture and use it to complete Table 12.13 without computation. Check by using the formula for f.

WORKED SOLUTION

(b)

Using the formula, check that the pattern holds for all $x \ge 1$ and $y \ge 1$.

WORKED SOLUTION

34.
Show that the function f does not have a limit at (0, 0) by examining the limits of f as $(x, y) \to (0, 0)$ along the line $y = x$ and along the parabola $y = x^2$:

$$f(x,y) = \frac{x^2 y}{x^4+y^2}, \quad (x,y) \neq (0,0).$$

()

35.
By approaching the origin along the positive x-axis and the positive y-axis, show that the following limit does not exist:

$$\lim_{(x,y)\to(0,0)} \frac{x+y^2}{2x+y}.$$

()

36.
Explain why the following function is not continuous along the line $y = 0$:

$$f(x,y) = \begin{cases} 1-x, & y \geq 0, \\ -2, & y < 0. \end{cases}$$

()

37.
A college admissions office uses the following equation to predict the grade point average of an incoming student:

$$z = 0.003x + 0.8y - 4,$$

()
where z is the predicted college GPA on a scale of 0 to 4.3, and x is the sum of the student's SAT Math and SAT Verbal on a scale of 400 to 1600, and y is the student's high school GPA on a scale of 0 to 4.3. The college admits students whose predicted GPA is at least 2.3.
(a)
Will a student with SATs of 1050 and high school GPA of 3.0 be admitted?
ANSWER
WORKED SOLUTION

(b)
Will every student with SATs of 1600 be admitted?
ANSWER
WORKED SOLUTION

(c)

Will every student with a high school GPA of 4.3 be admitted?

ANSWER

WORKED SOLUTION

(d)

Draw a contour diagram for the predicted GPA z with $400 \le x \le 1600$ and $0 \le y \le 4.3$. Shade the points corresponding to students who will be admitted.

WORKED SOLUTION

(e)

Which is more important, an extra 100 points on the SAT or an extra 0.5 of high school GPA?

ANSWER

WORKED SOLUTION

38.

By setting one variable constant, find a plane that intersects the graph of $z = (x^2 + 1) \sin y + xy^2$ in a:

(a)

Parabola

(b)

Straight line

(c)

Sine curve

39.

The temperature T (in °C) at any point in the region $-10 \le x \le 10$, $-10 \le y \le 10$ is given by the function

$$T(x, y) = 100 - x^2 - y^2.$$

()

(a)

Sketch isothermal curves (curves of constant temperature) for $T = 100$°C, $T = 75$°C, $T = 50$°C, $T = 25$°C, and $T = 0$°C.

ANSWER

(b)

A heat-seeking bug is put down at a point on the xy-plane. In which direction should it move to increase its temperature fastest? How is that direction related to the level curve through that point?

ANSWER

40.

Find a linear function whose graph is the plane that intersects the xy-plane along the line $y = 2x + 2$ and contains the point (1, 2, 2).

41.

(a)

Sketch the level curves of $z = \cos\ \sqrt{x^2 + y^2}$.

ANSWER

WORKED SOLUTION

(b)

Sketch a cross-section through the surface $z = \cos\ \sqrt{x^2 + y^2}$ in the plane containing the x- and z-axes. Put units on your axes.

ANSWER

WORKED SOLUTION

(c)

Sketch the cross-section through the surface $z = \cos\ \sqrt{x^2 + y^2}$ in the plane containing the z-axis and the line $y = x$ in the xy-plane.

ANSWER

WORKED SOLUTION

Problems 42-46 concern a vibrating guitar string. Snapshots of the guitar string at millisecond intervals are in Figure 12.107.

The guitar string is stretched tight along the x-axis from $x = 0$ to $x = \pi$. Each point on the string has an x-value, $0 \leq x \leq \pi$. As the string vibrates, each point on the string moves back and forth on either side of the x-axis. Let $y = f(x, t)$ be the displacement at time t of the point on the string located x units from the left end. A possible formula is

$$y = f(x, t) = \cos\ t\ \sin\ x, \quad 0 \leq x \leq \pi, \quad t \text{ in milliseconds.}$$

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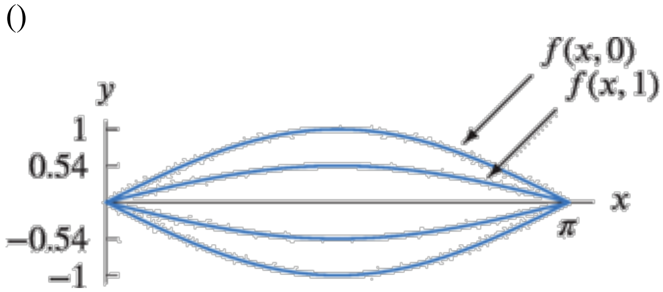

Figure 12.107: A vibrating guitar string: $f(x, t) = \cos t \sin x$ for four t values.

42.

Use the contour diagram for $f(x, t) = \cos t \sin x$ in Figure 12.108 to describe in words the cross-sections of f with t fixed and the cross-sections of f with x fixed. Explain what you see in terms of the vibrating string in Problems 43-46.

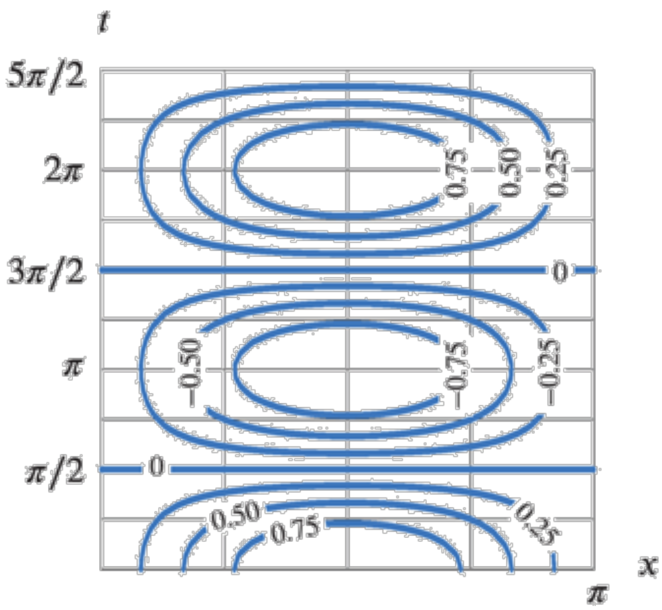

Figure 12.108

43.
Explain what the functions $f(x, 0)$ and $f(x, 1)$ represent in terms of the vibrating string.

44.
Explain what the functions $f(0, t)$ and $f(1, t)$ represent in terms of the vibrating string.

45.
(a)
Sketch graphs of y versus x for fixed t values, $t = 0, \pi/4, \pi/2, 3\pi/4, \pi$.

ANSWER ⊕

WORKED SOLUTION ⊕

(b)
Use your graphs to explain why this function could represent a vibrating guitar string.

WORKED SOLUTION ⊕

46.
Describe the motion of the guitar strings whose displacements are given by the following:
(a)
$y = g(x, t) = \cos 2t \sin x$

(b)
$y = h(x, t) = \cos t \sin 2x$

CAS Challenge Problems

47.
Let $A = (0, 0, 0)$ and $B = (2, 0, 0)$.
(a)

Find a point C in the xy-plane that is a distance 2 from both A and B.

ANSWER

(b)

Find a point D in 3-space that is a distance 2 from each of A, B, and C.

ANSWER

(c)

Describe the figure obtained by joining A, B, C, and D with straight lines.

ANSWER

48.

Let $f(x, y) = 3 + x + 2y$.

(a)

Find formulas for $f(x, f(x, y))$, $f(x, f(x, f(x, y)))$ by hand.

(b)

Consider $f(x, f(x, f(x, f(x, f(x, f(x, y))))))$. Conjecture a formula for this function and check your answer with a computer algebra system.

49.

A function $f(x, y, z)$ has the property that $f(1, 0, 1) = 20$, $f(1, 1, 1) = 16$, and $f(1, 1, 2) = 21$.

(a)

Estimate $f(1, 1, 3)$ and $f(1, 2, 1)$, assuming f is a linear function of each variable with the other variables held fixed.

ANSWER

WORKED SOLUTION

(b)

Suppose in fact that $f(x, y, z) = ax^2 + byz + czx^3 + d\,2^{x-y}$, for constants a, b, c and d. Which of your estimates in part a do you expect to be exact?

ANSWER

WORKED SOLUTION

(c)

Suppose in addition that $f(0, 0, 1) = 6$. Find an exact formula for f by solving for a, b, c, and d.

ANSWER

WORKED SOLUTION

(d)

Use the formula in part c to evaluate $f(1, 1, 3)$ and $f(1, 2, 1)$ exactly. Do the values confirm your answer to part b?

ANSWER

WORKED SOLUTION