

YOU MAY USE A CALCULATOR TO COMPUTE SOLUTIONS BUT SHOW YOUR SET-UPS.

Show all relevant work!

① You are standing above the point $(1, 3)$ on the surface $z = 20 - (2x^2 + y^2)$.

(a) In which direction should you walk to descend fastest? (Give your answer as a 2-vector.)

(b) If you start to move in this direction, what is the slope of your path?

(c) In what direction should you walk in order to remain at the same altitude? _____

② Consider the surface given by $z = g(x, y)$ shown below along with its contour diagram. At each point, A, B and C, on the contour diagram, indicate the direction of the gradient.

Then order the gradient magnitudes from least to greatest below.

_____ < _____ < _____

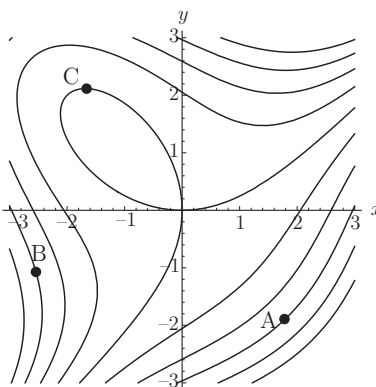


Figure 1: Level curves of $z = g(x, y)$.

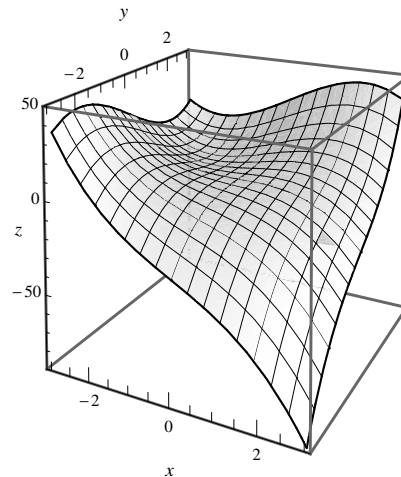


Figure 2: Surface $z = g(x, y)$.

③ Consider the surface $x^2 - \frac{y}{z^2} = 1$

(a) Verify the equation of the plane tangent to the surface at $(2, 3, 1)$ is $z = -\frac{2}{3}x + \frac{1}{6}y + \frac{11}{6}$.

(b) What is your rate of climb (or descent) as you move along the surface from the point $(2, 3, 1)$ in the direction $3\vec{i} + 4\vec{j}$? Are you climbing or descending?

④ The power, P (in watts) across a circuit is given by Watt's law: $P = I^2R$, where I is the current (in amps) flowing through the circuit and R is the resistance (in ohms). If we place two circuits, with resistance R_1 and R_2 , in parallel, then their combined resistance, R , is given by $\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$.

Suppose the current is 2 amps and increasing at 10^{-2} amp/sec and R_1 is 3 ohms and increasing at 0.5 ohm/sec, while R_2 is 5 ohms and decreasing at 0.1 ohm/sec.

Calculate the rate at which the wattage is changing.