Exam 3 Sections 7.6, 8.1, 8.3, 8.4 Spring 2005 Name:_____

Show all relevant work!

1. Locate and classify all critical points of the function $g(x, y) = 1 - x^2 + x + y^2 - y - 1$

2. Spacely Sprockets has just released its latest model, the Dominator. The demand function is $D(p) = 10,000 - 1000p$ sprockets each year when the price is p dollars. The supply function is $S(p) = 8000 + 1000p$ sprockets each year when the price is p dollars. This makes the equilibrium price \$1. The *Evans price adjustment model* assumes that if the price is set at a value other than the equlibrium price, it will change over time in such a way that its rate of change is proportional to the shortage $D(p) - S(p)$.

   (a) Write the differential equation given by the Evans price adjustment model for the price p as a function of time.

   (b) Find the general solution of the differential equation you wrote in part (a). (You will have two unknown constants, one being the constant of proportionality.)

   (c) Find the particular solution in which Dominator sprockets are initially priced at \$5 each but fall to \$3 each after 1 year.

3. Let $f(x, y) = x^4 y^2 - x$. Find the following, if possible:

(a) $\frac{\delta f}{\delta y}$

(b) $\frac{\delta f}{\delta x}\big|_{(1,-1)}$

(c) $\frac{\delta^2 f}{\delta x^2}$

(d) $\frac{\delta^2 f}{\delta y^2}$

(e) $\frac{\delta^2 f}{\delta x \delta y}$

(f) $\frac{\delta^2 f}{\delta y \delta x}$

(g) $\frac{\delta^2 f}{\delta x^2}\big|_{(1,-1)}$

(h) $\frac{\delta^2 f}{\delta y^2}\big|_{(1,-1)}$

(i) $\frac{\delta^2 f}{\delta x \delta y}\big|_{(1,-1)}$

(j) $\frac{\delta^2 f}{\delta y \delta x}\big|_{(1,-1)}$

4. Let $g(x, y, z) = 0.01x + 0.02y - 0.03z - 0.05$. Complete the following sentences.

    (a) g _____ by _____ units for every 1 unit of increase in z.

    (b) g _____ by _____ units for every 1 unit of increase in x.

    (c) _____ by 0.02 unit for every _____ .

5. For the given differential equation, find the particular solution indicated.
$\frac{dy}{dx} = \frac{y+1}{x}$; $y(1) = 2$

6. As marketing director for a bicycle manufacturer, you come up with the following scheme: You will offer to sell a dealer x bicycles and y tricycles for

$$R(x, y) = 3500 - 3500e^{-0.02x - 0.01y} \text{ dollars}$$

Find your marginal revenue for bicycles and tricycles. Are you likely to be fired for your suggestion?

7. Let $f(x,y) = xe^{xy}$. Find the following if the are defined:

(a) $\frac{\delta f}{\delta x}$

(b) $\frac{\delta f}{\delta y}$

(c) $\frac{\delta f}{\delta x}\Big|_{(1,-1)}$

(d) $\frac{\delta f}{\delta y}\Big|_{(1,-1)}$

(e) $\frac{\delta f}{\delta x}\Big|_{(-1,0)}$