Math 241

Exam 5 Chapter 5 Fall 2004 Name: Key

Show all relevant work!

If the problem is an application problem, you must set up the equations, but you may choose to solve them on the calculator. Just indicate that you have done so.

1. The consumer demand curve for Holiday Twinkies (red on the outside with green filling) is given by $q = (80 - p)^2$, $(0 \le p \le 80)$, where p is the price per case of twinkies and q is the demand in weekly sales. if twinkies cost \$20 per case to produce, at what price should twinkies be sold for the largest possible weekly profit? What will that profit be? (Round each to the nearest cent)

$R = p(80-p)^2$

$C = 20p = 20(80-p)^2$

$P = f(p) = (p-20)(80-p)^2 =$

$6400 - 160p + p^2$

$p - 20$

$p^3 + 320p - 20p^2 - 128000$

$6400p - 160p^2$

$f'(p) = (80-p)^2 - 2(p-20)(80-p)$

$f(p) = p^3 - 180p^2 + 9600p - 128$

$= (80-p)(80-p-2p+40)$

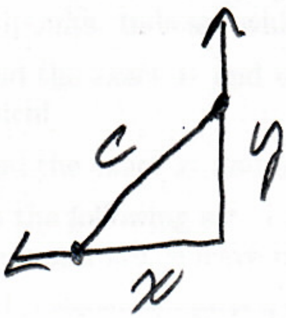
$= (80-p)(120-3p)$

$p = 80, 40$

\$40 per case for a profit of \$32000

3. A porche and BMW were driving away from the same intersection, the porche heading west, and the BMW heading north. The porche was travelling at 75 mph while the BMW was travelling at 50 mph. At a certain instant in time, the porche was one-half mile from the intersection while the BMW was one-fifth of a mile from it. How fast was their distance from each other changing at that instant?

Want $\left.\frac{dc}{dt}\right|_{(x,y)=(0.5,0.2)}$



$\frac{dx}{dt}=75,\ \frac{dy}{dt}=50$

$c^2=x^2+y^2 \Rightarrow c=\sqrt{x^2+y^2}$
$=\sqrt{0.5^2+0.2^2}\approx 0.54$

$2c\frac{dc}{dt}=2x\frac{dx}{dt}+2y\frac{dy}{dt}$

$\frac{dc}{dt}=\frac{(2(0.5)(75)+2(0.2)(50))}{2c}$

$\boxed{\approx 88.21 \text{ mph}}$

#3: Porche should be capitalized
and
Please round answer to the nearest 100th

4. (Worth 20 pts. The rest are worth 10 pts each.)
Consider the function $f(x) = 0.2(x+3)\sqrt{x}$ on [0,10]. Answer the following:

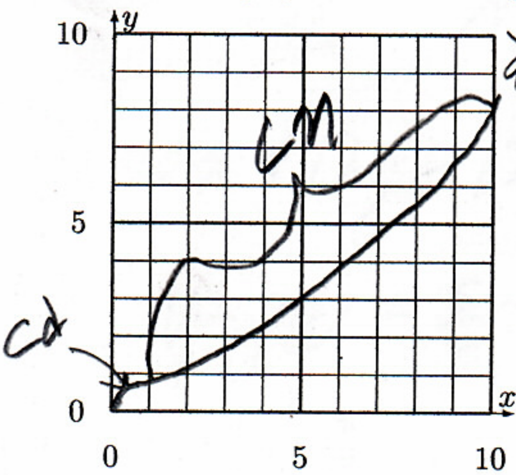
(a) Find simplified formulas for $f'(x)$ and $f''(x)$.

(b) Find the exact x- and y-coordinates of any stationary points, singular points and endpoints. Indicate which are which!

(c) Find the exact x- and y-coordinates of any absolute max's or min's. Indicate which are which!

(d) Find the exact x- and y-coordinates of any inflection points.

(e) On the following set of axes, sketch the graph of $f(x)$ indicating clearly which regions of the graph are concave up, and which are concave down.



a) $f(x) = 0.2(x^{\frac{3}{2}} + 3x^{\frac{1}{2}})$

$f'(x) = 0.2(\frac{3}{2}x^{\frac{1}{2}} + \frac{3}{2}x^{-\frac{1}{2}})$

$f''(x) = 0.2(\frac{3}{4}x^{-\frac{1}{2}} - \frac{3}{4}x^{-\frac{3}{2}})$

b) no stationary points
singular point at (0,0)
endpoints: (0,0) and $(10, 2.6\sqrt{10})$

c) abs. min at (0,0) abs. max at $(10, 2.6\sqrt{10})$

d) $\frac{1}{\sqrt{x}} = \frac{1}{\sqrt{x^3}}$. IP at (1, 0.8)

$x^3 = x$

$x = 1, \cancel{0}$

5. I want to fence in a rectangular apple orchard. The fencing for the east and west sides costs \$6 per foot, and the fencing for the north and south sides costs \$10 per foot. I have a budget of \$1200 for the project. What is the largest area I can enclose?

$A(x) = 1000x - \frac{5}{3}x^2$

$20x + 12y = 12000$

$A'(x) = 1000 - \frac{10}{3}x$

$y = 1000 - \frac{5}{3}x$

$x = 300$

150,000 sq. ft

\$12,000

6. Imagine you are in the video game system wholesale buisiness. The number of Nintendo Game Cubes you are prepared to supply to Video Game Emporium every week is given by $q = 0.2p^2 + 4p$ where p is the wholesale price it offers you. The Emporium is currently offering you \$60 per Game Cube. If the price it offers decreases at a rate of \$2 per week, how is the number you are prepared to supply going to change?

$$\frac{dq}{dt} = 0.4p \cdot \frac{dp}{dt} + 4\frac{dp}{dt}$$

$$= 0.4(60)(-2) + 4(-2) = -56$$

quantity decreasing at a rate of 56 systems / week