MATH 252: Calculus II Name: ________________

Test 1

1. On Mars, the acceleration due to gravity is 3.7 m/sec². A Martian stands on top of his spaceship 7.0 m above the surface of the moon. He leaps from the spaceship with an upward velocity of 2.5 m/sec. Find functions $a(t)$, $v(t)$, and $h(t)$ that give the Martian's acceleration, velocity and height above the surface of Mars t seconds after leaping from the spaceship. How long after leaping does the Martian reach his maximum height above the surface of Mars? What is the Martian's maximum height above the surface of Mars? How long does it take the Martian to land? With what velocity does the Martian land?

1 $a(t) = -3.7$

$v(t) = -3.7t + 2.5$

$h(t) = -1.85t^2 + 2.5t + 7.0$

2 $-3.7t + 2.5 = 0$

$t = \boxed{0.676 \text{ sec}}$

3 $h(0.676) = -1.85(0.676)^2 + 2.5(0.676) + 7.0 = \boxed{7.845 \text{ m}}$

4 $-1.85t^2 + 2.5t + 7.0 = 0$

$t = \cancel{-1.384}, \boxed{2.735 \text{ sec}}$

5 $v(2.735) = -3.7(2.735) + 2.5 = \boxed{-7.619 \text{ m/sec}}$

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Test 1

2. In Parts A and B, find the indefinite integral.

A. $\int \frac{x^2 - x - 1}{x}\, dx = \int x - 1 - \frac{1}{x}\, dx = \boxed{\frac{1}{2}x^2 - x - \ln|x| + C}$

B. $\int e^x + \sin x\, dx = \boxed{e^x - \cos x + C}$

In Parts C and D, find an function $F(x)$ such that $F'(x) = f(x)$ and $F(1) = 4$.

C. $f(x) = 3x^2 - 4x - 5$

$F(x) = x^3 - 2x^2 - 5x + C$

$F(1) = 4 = 1^3 - 2(1)^2 - 5(1) + C$

$\rightarrow C = 10$

$\boxed{F(x) = x^3 - 2x^2 - 5x + 10}$

D. $f(x) = (x+1)^3$

$F(x) = \frac{1}{4}(x+1)^4 + C$

$F(1) = 4 = \frac{1}{4}(1+1)^4 + C$

$\rightarrow C = 0$

$\boxed{F(x) = \frac{1}{4}(x+1)^4}$

In Part E, solve the initial-value problem.

E. $\frac{ds}{dt} = -32t + 100, \quad s(0) = 50$

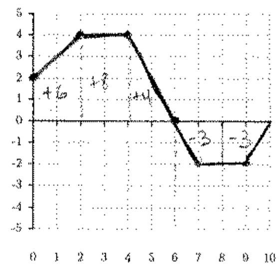
$s = -16t^2 + 100t + C$

$s(0) = 50 = -16(0)^2 + 100(0) + C$

$\rightarrow C = 50$

$\boxed{s = -16t^2 + 100t + 50}$

3. Let $f(0) = -8$, and consider the graph of $f'(x)$, the derivative of $f(x)$, shown below.



A. Find $\int_0^k f'(x)\,dx$ for $k = 0, 2, 4, 6, 8, 10$.

$\int_0^0 f'(x)\,dx = 0$ $\int_0^6 f'(x)\,dx = 18$

$\int_0^2 f'(x)\,dx = 6$ $\int_0^8 f'(x)\,dx = 15$

$\int_0^4 f'(x)\,dx = 14$ $\int_0^{10} f'(x)\,dx = 12$

B. Complete the table of values for $f(x)$.

x	0	2	4	6	8	10
$f(x)$	-8	-2	6	10	7	4
		-8+6	-8+14	-8+18	-8+15	-8+12

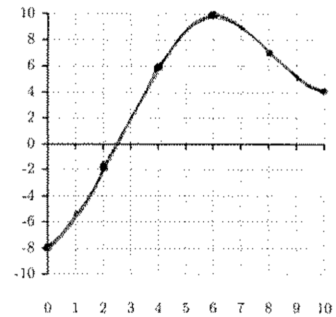
C. Find the maximum and minimum values of $f(x)$ for $0 \le x \le 10$.

local max at $x = 6$

f has a max of 10 at $x = 6$

f has a min of -8 at $x = 0$

D. Sketch the graph of $f(x)$ below.



E. Describe the concavity of $f(x)$.

$f(x)$ is concave up on $[0,2) \cup (9,10]$

$f(x)$ is concave down on $(4,7)$

$f(x)$ is neither concave up nor concave down on $[2,4] \cup [7,9]$.

4. In Parts A-C, find the indefinite integral.

A. $\int x(x^2+3)^2\,dx = \boxed{\frac{1}{6}(x^2+3)^3 + C}$

$u = x^2+3$

$du = 2x\,dx$

$\frac{1}{2}du = x\,dx$

$\frac{1}{2}\int u^2\,du = \frac{1}{2}\cdot\frac{1}{3}u^3 + C =$

B. $\int \frac{t}{1+t^2}\,dt = \boxed{\frac{1}{2}\ln(1+t^2) + C}$

$u = 1+t^2$

$du = 2t\,dt$

$\frac{1}{2}du = t\,dt$

$\frac{1}{2}\int \frac{1}{u}\,du = \frac{1}{2}\ln|u| + C =$

C. $\int \frac{e^{\sqrt{x}}}{\sqrt{x}}\,dx = \boxed{2e^{\sqrt{x}} + C}$

$u = \sqrt{x} = x^{1/2}$

$du = \frac{1}{2}x^{-1/2} = \frac{1}{2\sqrt{x}}\,dx$

$2\,du = \frac{1}{\sqrt{x}}\,dx$

$2\int e^u\,du = 2e^u + C =$

In Part D, use the Fundamental Theorem of Calculus to calculate the definite integral.

D. $\int_1^4 \frac{\cos\sqrt{x}}{\sqrt{x}}\,dx = 2\int_1^2 \cos u\,du = 2\sin u\Big|_1^2 = \boxed{2\sin 2 - 2\sin 1}$

radians

$\approx \boxed{0.136}$

$u = \sqrt{x}$

$du = \frac{1}{2\sqrt{x}}\,dx$

$2\,du = \frac{1}{\sqrt{x}}\,dx$

$x = 1 \rightarrow u = \sqrt{1} = 1$

$x = 4 \rightarrow u = \sqrt{4} = 2$

5. A jet needs to be moving at 195 mph (286 ft/sec) in order to take off. It takes 22 seconds for the jet to accelerate from 0 to 195 mph. Assuming constant acceleration, what is the minimum length of the runway?

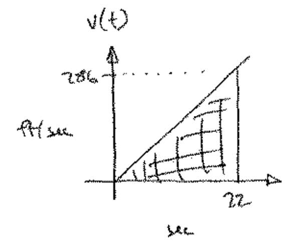
$a(t) = \frac{\Delta v}{\Delta t} = \frac{286}{22} = 13\text{ ft/sec}^2$

$v(t) = 13t$

$s(t) = 6.5t^2$

$s(22) = 6.5(22)^2 = \boxed{3146\text{ ft}}$

Alt



$\text{Area} = \frac{1}{2}(22)(286) = \boxed{3146\text{ ft}}$