# 1 Linear Equations in Linear Algebra

## 1.1 SOLUTIONS

**Notes:** The key exercises are 7 (or 11 or 12), 19–22, and 25. For brevity, the symbols R1, R2,…, stand for row 1 (or equation 1), row 2 (or equation 2), and so on. Additional notes are at the end of the section.

**1.** $\begin{aligned} x_1 + 5x_2 &= 7 \\ -2x_1 - 7x_2 &= -5 \end{aligned}$ $\quad \begin{bmatrix} 1 & 5 & 7 \\ -2 & -7 & -5 \end{bmatrix}$

Replace R2 by R2 + (2)R1 and obtain: $\begin{aligned} x_1 + 5x_2 &= 7 \\ 3x_2 &= 9 \end{aligned}$ $\quad \begin{bmatrix} 1 & 5 & 7 \\ 0 & 3 & 9 \end{bmatrix}$

Scale R2 by 1/3: $\begin{aligned} x_1 + 5x_2 &= 7 \\ x_2 &= 3 \end{aligned}$ $\quad \begin{bmatrix} 1 & 5 & 7 \\ 0 & 1 & 3 \end{bmatrix}$

Replace R1 by R1 + (–5)R2: $\begin{aligned} x_1 \quad &= -8 \\ x_2 &= 3 \end{aligned}$ $\quad \begin{bmatrix} 1 & 0 & -8 \\ 0 & 1 & 3 \end{bmatrix}$

The solution is $(x_1, x_2) = (-8, 3)$, or simply $(-8, 3)$.

**2.** $\begin{aligned} 2x_1 + 4x_2 &= -4 \\ 5x_1 + 7x_2 &= 11 \end{aligned}$ $\quad \begin{bmatrix} 2 & 4 & -4 \\ 5 & 7 & 11 \end{bmatrix}$

Scale R1 by 1/2 and obtain: $\begin{aligned} x_1 + 2x_2 &= -2 \\ 5x_1 + 7x_2 &= 11 \end{aligned}$ $\quad \begin{bmatrix} 1 & 2 & -2 \\ 5 & 7 & 11 \end{bmatrix}$

Replace R2 by R2 + (–5)R1: $\begin{aligned} x_1 + 2x_2 &= -2 \\ -3x_2 &= 21 \end{aligned}$ $\quad \begin{bmatrix} 1 & 2 & -2 \\ 0 & -3 & 21 \end{bmatrix}$

Scale R2 by –1/3: $\begin{aligned} x_1 + 2x_2 &= -2 \\ x_2 &= -7 \end{aligned}$ $\quad \begin{bmatrix} 1 & 2 & -2 \\ 0 & 1 & -7 \end{bmatrix}$

Replace R1 by R1 + (–2)R2: $\begin{aligned} x_1 \quad &= 12 \\ x_2 &= -7 \end{aligned}$ $\quad \begin{bmatrix} 1 & 0 & 12 \\ 0 & 1 & -7 \end{bmatrix}$

The solution is $(x_1, x_2) = (12, -7)$, or simply $(12, -7)$.

**3.** The point of intersection satisfies the system of two linear equations:

$$\begin{aligned} x_1 + 5x_2 &= 7 \\ x_1 - 2x_2 &= -2 \end{aligned} \qquad \begin{bmatrix} 1 & 5 & 7 \\ 1 & -2 & -2 \end{bmatrix}$$

Replace R2 by R2 + (–1)R1 and obtain:

$$\begin{aligned} x_1 + 5x_2 &= 7 \\ -7x_2 &= -9 \end{aligned} \qquad \begin{bmatrix} 1 & 5 & 7 \\ 0 & -7 & -9 \end{bmatrix}$$

Scale R2 by –1/7:

$$\begin{aligned} x_1 + 5x_2 &= 7 \\ x_2 &= 9/7 \end{aligned} \qquad \begin{bmatrix} 1 & 5 & 7 \\ 0 & 1 & 9/7 \end{bmatrix}$$

Replace R1 by R1 + (–5)R2:

$$\begin{aligned} x_1 \qquad &= 4/7 \\ x_2 &= 9/7 \end{aligned} \qquad \begin{bmatrix} 1 & 0 & 4/7 \\ 0 & 1 & 9/7 \end{bmatrix}$$

The point of intersection is $(x_1, x_2) = (4/7, 9/7)$.

**4.** The point of intersection satisfies the system of two linear equations:

$$\begin{aligned} x_1 - 5x_2 &= 1 \\ 3x_1 - 7x_2 &= 5 \end{aligned} \qquad \begin{bmatrix} 1 & -5 & 1 \\ 3 & -7 & 5 \end{bmatrix}$$

Replace R2 by R2 + (–3)R1 and obtain:

$$\begin{aligned} x_1 - 5x_2 &= 1 \\ 8x_2 &= 2 \end{aligned} \qquad \begin{bmatrix} 1 & -5 & 1 \\ 0 & 8 & 2 \end{bmatrix}$$

Scale R2 by 1/8:

$$\begin{aligned} x_1 - 5x_2 &= 1 \\ x_2 &= 1/4 \end{aligned} \qquad \begin{bmatrix} 1 & -5 & 1 \\ 0 & 1 & 1/4 \end{bmatrix}$$

Replace R1 by R1 + (5)R2:

$$\begin{aligned} x_1 \qquad &= 9/4 \\ x_2 &= 1/4 \end{aligned} \qquad \begin{bmatrix} 1 & 0 & 9/4 \\ 0 & 1 & 1/4 \end{bmatrix}$$

The point of intersection is $(x_1, x_2) = (9/4, 1/4)$.

**5.** The system is already in "triangular" form. The fourth equation is $x_4 = -5$, and the other equations do not contain the variable $x_4$. The next two steps should be to use the variable $x_3$ in the third equation to eliminate that variable from the first two equations. In matrix notation, that means to replace R2 by its sum with 3 times R3, and then replace R1 by its sum with –5 times R3.

**6.** One more step will put the system in triangular form. Replace R4 by its sum with –3 times R3, which produces $\begin{bmatrix} 1 & -6 & 4 & 0 & -1 \\ 0 & 2 & -7 & 0 & 4 \\ 0 & 0 & 1 & 2 & -3 \\ 0 & 0 & 0 & -5 & 15 \end{bmatrix}$. After that, the next step is to scale the fourth row by –1/5.

**7.** Ordinarily, the next step would be to interchange R3 and R4, to put a 1 in the third row and third column. But in this case, the third row of the augmented matrix corresponds to the equation $0\,x_1 + 0\,x_2 + 0\,x_3 = 1$, or simply, $0 = 1$. A system containing this condition has no solution. Further row operations are unnecessary once an equation such as $0 = 1$ is evident.
The solution set is empty.

**8**. The standard row operations are:

$$\begin{bmatrix}1 & -4 & 9 & 0\\0 & 1 & 7 & 0\\0 & 0 & 2 & 0\end{bmatrix} \sim \begin{bmatrix}1 & -4 & 9 & 0\\0 & 1 & 7 & 0\\0 & 0 & 1 & 0\end{bmatrix} \sim \begin{bmatrix}1 & -4 & 0 & 0\\0 & 1 & 0 & 0\\0 & 0 & 1 & 0\end{bmatrix} \sim \begin{bmatrix}1 & 0 & 0 & 0\\0 & 1 & 0 & 0\\0 & 0 & 1 & 0\end{bmatrix}$$

The solution set contains one solution: (0, 0, 0).

**9**. The system has already been reduced to triangular form. Begin by scaling the fourth row by 1/2 and then replacing R3 by R3 + (3)R4:

$$\begin{bmatrix}1 & -1 & 0 & 0 & -4\\0 & 1 & -3 & 0 & -7\\0 & 0 & 1 & -3 & -1\\0 & 0 & 0 & 2 & 4\end{bmatrix} \sim \begin{bmatrix}1 & -1 & 0 & 0 & -4\\0 & 1 & -3 & 0 & 7\\0 & 0 & 1 & -3 & -1\\0 & 0 & 0 & 1 & 2\end{bmatrix} \sim \begin{bmatrix}1 & -1 & 0 & 0 & -4\\0 & 1 & -3 & 0 & -7\\0 & 0 & 1 & 0 & 5\\0 & 0 & 0 & 1 & 2\end{bmatrix}$$

Next, replace R2 by R2 + (3)R3. Finally, replace R1 by R1 + R2:

$$\sim \begin{bmatrix}1 & -1 & 0 & 0 & -4\\0 & 1 & 0 & 0 & 8\\0 & 0 & 1 & 0 & 5\\0 & 0 & 0 & 1 & 2\end{bmatrix} \sim \begin{bmatrix}1 & 0 & 0 & 0 & 4\\0 & 1 & 0 & 0 & 8\\0 & 0 & 1 & 0 & 5\\0 & 0 & 0 & 1 & 2\end{bmatrix}$$

The solution set contains one solution: (4, 8, 5, 2).

**10**. The system has already been reduced to triangular form. Use the 1 in the fourth row to change the –4 and 3 above it to zeros. That is, replace R2 by R2 + (4)R4 and replace R1 by R1 + (–3)R4. For the final step, replace R1 by R1 + (2)R2.

$$\begin{bmatrix}1 & -2 & 0 & 3 & -2\\0 & 1 & 0 & -4 & 7\\0 & 0 & 1 & 0 & 6\\0 & 0 & 0 & 1 & -3\end{bmatrix} \sim \begin{bmatrix}1 & -2 & 0 & 0 & 7\\0 & 1 & 0 & 0 & -5\\0 & 0 & 1 & 0 & 6\\0 & 0 & 0 & 1 & -3\end{bmatrix} \sim \begin{bmatrix}1 & 0 & 0 & 0 & -3\\0 & 1 & 0 & 0 & -5\\0 & 0 & 1 & 0 & 6\\0 & 0 & 0 & 1 & -3\end{bmatrix}$$

The solution set contains one solution: (–3, –5, 6, –3).

**11**. First, swap R1 and R2. Then replace R3 by R3 + (–3)R1. Finally, replace R3 by R3 + (2)R2.

$$\begin{bmatrix}0 & 1 & 4 & -5\\1 & 3 & 5 & -2\\3 & 7 & 7 & 6\end{bmatrix} \sim \begin{bmatrix}1 & 3 & 5 & -2\\0 & 1 & 4 & -5\\3 & 7 & 7 & 6\end{bmatrix} \sim \begin{bmatrix}1 & 3 & 5 & -2\\0 & 1 & 4 & -5\\0 & -2 & -8 & 12\end{bmatrix} \sim \begin{bmatrix}1 & 3 & 5 & -2\\0 & 1 & 4 & -5\\0 & 0 & 0 & 2\end{bmatrix}$$

The system is inconsistent, because the last row would require that 0 = 2 if there were a solution. The solution set is empty.

**12**. Replace R2 by R2 + (–3)R1 and replace R3 by R3 + (4)R1. Finally, replace R3 by R3 + (3)R2.

$$\begin{bmatrix}1 & -3 & 4 & -4\\3 & -7 & 7 & -8\\-4 & 6 & -1 & 7\end{bmatrix} \sim \begin{bmatrix}1 & -3 & 4 & -4\\0 & 2 & -5 & 4\\0 & -6 & 15 & -9\end{bmatrix} \sim \begin{bmatrix}1 & -3 & 4 & -4\\0 & 2 & -5 & 4\\0 & 0 & 0 & 3\end{bmatrix}$$

The system is inconsistent, because the last row would require that 0 = 3 if there were a solution. The solution set is empty.

**13.** $\begin{bmatrix} 1 & 0 & -3 & 8 \\ 2 & 2 & 9 & 7 \\ 0 & 1 & 5 & -2 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & -3 & 8 \\ 0 & 2 & 15 & -9 \\ 0 & 1 & 5 & -2 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & -3 & 8 \\ 0 & 1 & 5 & -2 \\ 0 & 2 & 15 & -9 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & -3 & 8 \\ 0 & 1 & 5 & -2 \\ 0 & 0 & 5 & -5 \end{bmatrix}$

$\sim \begin{bmatrix} 1 & 0 & -3 & 8 \\ 0 & 1 & 5 & -2 \\ 0 & 0 & 1 & -1 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 0 & 5 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & -1 \end{bmatrix}$. The solution is (5, 3, –1).

**14.** $\begin{bmatrix} 1 & -3 & 0 & 5 \\ -1 & 1 & 5 & 2 \\ 0 & 1 & 1 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & -3 & 0 & 5 \\ 0 & -2 & 5 & 7 \\ 0 & 1 & 1 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & -3 & 0 & 5 \\ 0 & 1 & 1 & 0 \\ 0 & -2 & 5 & 7 \end{bmatrix} \sim \begin{bmatrix} 1 & -3 & 0 & 5 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 7 & 7 \end{bmatrix}$

$\sim \begin{bmatrix} 1 & -3 & 0 & 5 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix} \sim \begin{bmatrix} 1 & -3 & 0 & 5 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 1 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 1 \end{bmatrix}$. The solution is (2, –1, 1).

**15.** First, replace R4 by R4 + (–3)R1, then replace R3 by R3 + (2)R2, and finally replace R4 by R4 + (3)R3.

$$\begin{bmatrix} 1 & 0 & 3 & 0 & 2 \\ 0 & 1 & 0 & -3 & 3 \\ 0 & -2 & 3 & 2 & 1 \\ 3 & 0 & 0 & 7 & -5 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 3 & 0 & 2 \\ 0 & 1 & 0 & -3 & 3 \\ 0 & -2 & 3 & 2 & 1 \\ 0 & 0 & -9 & 7 & -11 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & 0 & 3 & 0 & 2 \\ 0 & 1 & 0 & -3 & 3 \\ 0 & 0 & 3 & -4 & 7 \\ 0 & 0 & -9 & 7 & -11 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 3 & 0 & 2 \\ 0 & 1 & 0 & -3 & 3 \\ 0 & 0 & 3 & -4 & 7 \\ 0 & 0 & 0 & -5 & 10 \end{bmatrix}$$

The resulting triangular system indicates that a solution exists. In fact, using the argument from Example 2, one can see that the solution is unique.

**16.** First replace R4 by R4 + (2)R1 and replace R4 by R4 + (–3/2)R2. (One could also scale R2 before adding to R4, but the arithmetic is rather easy keeping R2 unchanged.) Finally, replace R4 by R4 + R3.

$$\begin{bmatrix} 1 & 0 & 0 & -2 & -3 \\ 0 & 2 & 2 & 0 & 0 \\ 0 & 0 & 1 & 3 & 1 \\ -2 & 3 & 2 & 1 & 5 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 0 & -2 & -3 \\ 0 & 2 & 2 & 0 & 0 \\ 0 & 0 & 1 & 3 & 1 \\ 0 & 3 & 2 & -3 & -1 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & 0 & 0 & -2 & -3 \\ 0 & 2 & 2 & 0 & 0 \\ 0 & 0 & 1 & 3 & 1 \\ 0 & 0 & -1 & -3 & -1 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 0 & -2 & -3 \\ 0 & 2 & 2 & 0 & 0 \\ 0 & 0 & 1 & 3 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

The system is now in triangular form and has a solution. The next section discusses how to continue with this type of system.

**17.** Row reduce the augmented matrix corresponding to the given system of three equations:

$$\begin{bmatrix} 1 & -4 & 1 \\ 2 & -1 & -3 \\ -1 & -3 & 4 \end{bmatrix} \sim \begin{bmatrix} 1 & -4 & 1 \\ 0 & 7 & -5 \\ 0 & -7 & 5 \end{bmatrix} \sim \begin{bmatrix} 1 & -4 & 1 \\ 0 & 7 & -5 \\ 0 & 0 & 0 \end{bmatrix}$$

The system is consistent, and using the argument from Example 2, there is only one solution. So the three lines have only one point in common.

**18.** Row reduce the augmented matrix corresponding to the given system of three equations:

$$\begin{bmatrix} 1 & 2 & 1 & 4 \\ 0 & 1 & -1 & 1 \\ 1 & 3 & 0 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & 2 & 1 & 4 \\ 0 & 1 & -1 & 1 \\ 0 & 1 & -1 & -4 \end{bmatrix} \sim \begin{bmatrix} 1 & 2 & 1 & 4 \\ 0 & 1 & -1 & 1 \\ 0 & 0 & 0 & -5 \end{bmatrix}$$

The third equation, $0 = -5$, shows that the system is inconsistent, so the three planes have no point in common.

**19.** $\begin{bmatrix} 1 & h & 4 \\ 3 & 6 & 8 \end{bmatrix} \sim \begin{bmatrix} 1 & h & 4 \\ 0 & 6-3h & -4 \end{bmatrix}$ Write $c$ for $6 - 3h$. If $c = 0$, that is, if $h = 2$, then the system has no solution, because 0 cannot equal –4. Otherwise, when $h \neq 2$, the system has a solution.

**20.** $\begin{bmatrix} 1 & h & -3 \\ -2 & 4 & 6 \end{bmatrix} \sim \begin{bmatrix} 1 & h & -3 \\ 0 & 4+2h & 0 \end{bmatrix}$. Write $c$ for $4 + 2h$. Then the second equation $cx_2 = 0$ has a solution for every value of c. So the system is consistent for all $h$.

**21.** $\begin{bmatrix} 1 & 3 & -2 \\ -4 & h & 8 \end{bmatrix} \sim \begin{bmatrix} 1 & 3 & -2 \\ 0 & h+12 & 0 \end{bmatrix}$. Write $c$ for $h + 12$. Then the second equation $cx_2 = 0$ has a solution for every value of $c$. So the system is consistent for all $h$.

**22.** $\begin{bmatrix} 2 & -3 & h \\ -6 & 9 & 5 \end{bmatrix} \sim \begin{bmatrix} 2 & -3 & h \\ 0 & 0 & 5+3h \end{bmatrix}$. The system is consistent if and only if $5 + 3h = 0$, that is, if and only if $h = -5/3$.

**23.** **a.** True. See the remarks following the box titled *Elementary Row Operations.*

**b.** False. A $5 \times 6$ matrix has five rows.

**c.** False. The description given applied to a single solution. The solution *set* consists of all possible solutions. Only in special cases does the solution set consist of exactly one solution. Mark a statement True only if the statement is *always* true.

**d.** True. See the box before Example 2.

**24.** **a.** True. See the box preceding the subsection titled *Existence and Uniqueness Questions.*

**b.** False. The definition of *row equivalent* requires that there exist a sequence of row operations that transforms one matrix into the other.

**c.** False. By definition, an inconsistent system has *no* solution.

**d.** True. This definition of *equivalent systems* is in the second paragraph after equation (2).

**25.** $\begin{bmatrix} 1 & -4 & 7 & g \\ 0 & 3 & -5 & h \\ -2 & 5 & -9 & k \end{bmatrix} \sim \begin{bmatrix} 1 & -4 & 7 & g \\ 0 & 3 & -5 & h \\ 0 & -3 & 5 & k+2g \end{bmatrix} \sim \begin{bmatrix} 1 & -4 & 7 & g \\ 0 & 3 & -5 & h \\ 0 & 0 & 0 & k+2g+h \end{bmatrix}$

Let $b$ denote the number $k + 2g + h$. Then the third equation represented by the augmented matrix above is $0 = b$. This equation is possible if and only if $b$ is zero. So the original system has a solution if and only if $k + 2g + h = 0$.

**26.** A basic principle of this section is that row operations do not affect the solution set of a linear system. Begin with a simple augmented matrix for which the solution is obviously (–2, 1, 0), and then perform any elementary row operations to produce other augmented matrices. Here are three examples. The fact that they are all row equivalent proves that they all have the solution set (–2, 1, 0).

$$\begin{bmatrix} 1 & 0 & 0 & -2 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 0 & -2 \\ 2 & 1 & 0 & -3 \\ 0 & 0 & 1 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 0 & -2 \\ 2 & 1 & 0 & -3 \\ 2 & 0 & 1 & -4 \end{bmatrix}$$

**27.** Study the augmented matrix for the given system, replacing R2 by R2 + (–$c$)R1:

$$\begin{bmatrix} 1 & 3 & f \\ c & d & g \end{bmatrix} \sim \begin{bmatrix} 1 & 3 & f \\ 0 & d-3c & g-cf \end{bmatrix}$$

This shows that shows $d - 3c$ must be nonzero, since $f$ and $g$ are arbitrary. Otherwise, for some choices of $f$ and $g$ the second row would correspond to an equation of the form $0 = b$, where $b$ is nonzero. Thus $d \neq 3c$.

**28.** Row reduce the augmented matrix for the given system. Scale the first row by $1/a$, which is possible since $a$ is nonzero. Then replace R2 by R2 + (–$c$)R1.

$$\begin{bmatrix} a & b & f \\ c & d & g \end{bmatrix} \sim \begin{bmatrix} 1 & b/a & f/a \\ c & d & g \end{bmatrix} \sim \begin{bmatrix} 1 & b/a & f/a \\ 0 & d-c(b/a) & g-c(f/a) \end{bmatrix}$$

The quantity $d - c(b/a)$ must be nonzero, in order for the system to be consistent when the quantity $g - c(f/a)$ is nonzero (which can certainly happen). The condition that $d - c(b/a) \neq 0$ can also be written as $ad - bc \neq 0$, or $ad \neq bc$.

**29.** Swap R1 and R2; swap R1 and R2.

**30.** Multiply R2 by –1/2; multiply R2 by –2.

**31.** Replace R3 by R3 + (–4)R1; replace R3 by R3 + (4)R1.

**32.** Replace R3 by R3 + (3)R2; replace R3 by R3 + (–3)R2.

**33.** The first equation was given. The others are:

$$T_2 = (T_1 + 20 + 40 + T_3)/4, \quad \text{or} \quad 4T_2 - T_1 - T_3 = 60$$
$$T_3 = (T_4 + T_2 + 40 + 30)/4, \quad \text{or} \quad 4T_3 - T_4 - T_2 = 70$$
$$T_4 = (10 + T_1 + T_3 + 30)/4, \quad \text{or} \quad 4T_4 - T_1 - T_3 = 40$$

Rearranging,

$$\begin{array}{rcrcrcrcl} 4T_1 & - & T_2 & & & - & T_4 & = & 30 \\ -T_1 & + & 4T_2 & - & T_3 & & & = & 60 \\ & & -T_2 & + & 4T_3 & - & T_4 & = & 70 \\ -T_1 & & & - & T_3 & + & 4T_4 & = & 40 \end{array}$$

**34.** Begin by interchanging R1 and R4, then create zeros in the first column:

$$\begin{bmatrix} 4 & -1 & 0 & -1 & 30 \\ -1 & 4 & -1 & 0 & 60 \\ 0 & -1 & 4 & -1 & 70 \\ -1 & 0 & -1 & 4 & 40 \end{bmatrix} \sim \begin{bmatrix} -1 & 0 & -1 & 4 & 40 \\ -1 & 4 & -1 & 0 & 60 \\ 0 & -1 & 4 & -1 & 70 \\ 4 & -1 & 0 & -1 & 30 \end{bmatrix} \sim \begin{bmatrix} -1 & 0 & -1 & 4 & 40 \\ 0 & 4 & 0 & -4 & 20 \\ 0 & -1 & 4 & -1 & 70 \\ 0 & -1 & -4 & 15 & 190 \end{bmatrix}$$

Scale R1 by –1 and R2 by 1/4, create zeros in the second column, and replace R4 by R4 + R3:

$$\sim \begin{bmatrix} 1 & 0 & 1 & -4 & -40 \\ 0 & 1 & 0 & -1 & 5 \\ 0 & -1 & 4 & -1 & 70 \\ 0 & -1 & -4 & 15 & 190 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 1 & -4 & -40 \\ 0 & 1 & 0 & -1 & 5 \\ 0 & 0 & 4 & -2 & 75 \\ 0 & 0 & -4 & 14 & 195 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 1 & -4 & -40 \\ 0 & 1 & 0 & -1 & 5 \\ 0 & 0 & 4 & -2 & 75 \\ 0 & 0 & 0 & 12 & 270 \end{bmatrix}$$

Scale R4 by 1/12, use R4 to create zeros in column 4, and then scale R3 by 1/4:

$$\sim \begin{bmatrix} 1 & 0 & 1 & -4 & -40 \\ 0 & 1 & 0 & -1 & 5 \\ 0 & 0 & 4 & -2 & 75 \\ 0 & 0 & 0 & 1 & 22.5 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 1 & 0 & 50 \\ 0 & 1 & 0 & 0 & 27.5 \\ 0 & 0 & 4 & 0 & 120 \\ 0 & 0 & 0 & 1 & 22.5 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 1 & 0 & 50 \\ 0 & 1 & 0 & 0 & 27.5 \\ 0 & 0 & 1 & 0 & 30 \\ 0 & 0 & 0 & 1 & 22.5 \end{bmatrix}$$

The last step is to replace R1 by R1 + (–1)R3:

$$\sim \begin{bmatrix} 1 & 0 & 0 & 0 & 20.0 \\ 0 & 1 & 0 & 0 & 27.5 \\ 0 & 0 & 1 & 0 & 30.0 \\ 0 & 0 & 0 & 1 & 22.5 \end{bmatrix}.$$ The solution is (20, 27.5, 30, 22.5).

**Notes**: The *Study Guide* includes a "Mathematical Note" about statements, "If … , then … ."

This early in the course, students typically use single row operations to reduce a matrix. As a result, even the small grid for Exercise 34 leads to about 25 multiplications or additions (not counting operations with zero). This exercise should give students an appreciation for matrix programs such as MATLAB. Exercise 14 in Section 1.10 returns to this problem and states the solution in case students have not already solved the system of equations. Exercise 31 in Section 2.5 uses this same type of problem in connection with an LU factorization.

For instructors who wish to use technology in the course, the *Study Guide* provides boxed MATLAB notes at the ends of many sections. Parallel notes for Maple, Mathematica, and the TI-83+/86/89 and HP-48G calculators appear in separate appendices at the end of the *Study Guide*. The MATLAB box for Section 1.1 describes how to access the data that is available for all numerical exercises in the text. This feature has the ability to save students time if they regularly have their matrix program at hand when studying linear algebra. The MATLAB box also explains the basic commands `replace`, `swap`, and `scale`. These commands are included in the text data sets, available from the text web site, www.laylinalgebra.com.