1.2 SOLUTIONS

Notes: The key exercises are 1–20 and 23–28. (Students should work at least four or five from Exercises 7–14, in preparation for Section 1.5.)

1. Reduced echelon form: a and b. Echelon form: d. Not echelon: c.

2. Reduced echelon form: a. Echelon form: b and d. Not echelon: c.

3. $$\begin{bmatrix}1&2&3&4\\4&5&6&7\\6&7&8&9\end{bmatrix}\sim\begin{bmatrix}1&2&3&4\\0&-3&-6&-9\\0&-5&-10&-15\end{bmatrix}\sim\begin{bmatrix}1&2&3&4\\0&1&2&3\\0&-5&-10&-15\end{bmatrix}$$

$$\sim\begin{bmatrix}1&2&3&4\\0&1&2&3\\0&0&0&0\end{bmatrix}\sim\begin{bmatrix}\textcircled{1}&0&-1&-2\\0&\textcircled{1}&2&3\\0&0&0&0\end{bmatrix}.$$ Pivot cols 1 and 2. $$\begin{bmatrix}\textcircled{1}&2&3&4\\4&\textcircled{5}&6&7\\6&7&8&9\end{bmatrix}$$

4. $$\begin{bmatrix}1&3&5&7\\3&5&7&9\\5&7&9&1\end{bmatrix}\sim\begin{bmatrix}1&3&5&7\\0&-4&-8&-12\\0&-8&-16&-34\end{bmatrix}\sim\begin{bmatrix}1&3&5&7\\0&1&2&3\\0&-8&-16&-34\end{bmatrix}\sim\begin{bmatrix}1&3&5&7\\0&1&2&3\\0&0&0&-10\end{bmatrix}$$

$$\sim\begin{bmatrix}1&3&5&7\\0&1&2&3\\0&0&0&1\end{bmatrix}\sim\begin{bmatrix}1&3&5&0\\0&1&2&0\\0&0&0&1\end{bmatrix}\sim\begin{bmatrix}\textcircled{1}&0&-1&0\\0&\textcircled{1}&2&0\\0&0&0&\textcircled{1}\end{bmatrix}.$$ Pivot cols 1, 2, and 4 $$\begin{bmatrix}\textcircled{1}&3&5&7\\3&\textcircled{5}&7&9\\5&7&9&\textcircled{1}\end{bmatrix}$$

5. $$\begin{bmatrix}\blacksquare&*\\0&\blacksquare\end{bmatrix},\begin{bmatrix}\blacksquare&*\\0&0\end{bmatrix},\begin{bmatrix}0&\blacksquare\\0&0\end{bmatrix}$$

6. $$\begin{bmatrix}\blacksquare&*\\0&\blacksquare\\0&0\end{bmatrix},\begin{bmatrix}\blacksquare&*\\0&0\\0&0\end{bmatrix},\begin{bmatrix}0&\blacksquare\\0&0\\0&0\end{bmatrix}$$

7. $$\begin{bmatrix}1&3&4&7\\3&9&7&6\end{bmatrix}\sim\begin{bmatrix}1&3&4&7\\0&0&-5&-15\end{bmatrix}\sim\begin{bmatrix}1&3&4&7\\0&0&1&3\end{bmatrix}\sim\begin{bmatrix}\textcircled{1}&3&0&-5\\0&0&\textcircled{1}&3\end{bmatrix}$$

Corresponding system of equations: $$\begin{aligned}\textcircled{x_1} + 3x_2 &= -5\\ \textcircled{x_3} &= 3\end{aligned}$$

The basic variables (corresponding to the pivot positions) are x_1 and x_3. The remaining variable x_2 is free. Solve for the basic variables in terms of the free variable. The general solution is

$$\begin{cases}x_1 = -5 - 3x_2\\ x_2 \text{ is free}\\ x_3 = 3\end{cases}$$

Note: Exercise 7 is paired with Exercise 10.

8. $\begin{bmatrix} 1 & 4 & 0 & 7 \\ 2 & 7 & 0 & 10 \end{bmatrix} \sim \begin{bmatrix} 1 & 4 & 0 & 7 \\ 0 & -1 & 0 & -4 \end{bmatrix} \sim \begin{bmatrix} 1 & 4 & 0 & 7 \\ 0 & 1 & 0 & 4 \end{bmatrix} \sim \begin{bmatrix} \textcircled{1} & 0 & 0 & -9 \\ 0 & \textcircled{1} & 0 & 4 \end{bmatrix}$

Corresponding system of equations:
$$\begin{aligned} \textcircled{x_1} \quad &= -9 \\ \textcircled{x_2} \quad &= 4 \end{aligned}$$

The basic variables (corresponding to the pivot positions) are x_1 and x_2. The remaining variable x_3 is free. Solve for the basic variables in terms of the free variable. In this particular problem, the basic variables do not depend on the value of the free variable.

General solution: $\begin{cases} x_1 = -9 \\ x_2 = 4 \\ x_3 \text{ is free} \end{cases}$

Note: A common error in Exercise 8 is to assume that x_3 is zero. To avoid this, identify the basic variables first. Any remaining variables are *free*. (This type of computation will arise in Chapter 5.)

9. $\begin{bmatrix} 0 & 1 & -6 & 5 \\ 1 & -2 & 7 & -6 \end{bmatrix} \sim \begin{bmatrix} 1 & -2 & 7 & -6 \\ 0 & 1 & -6 & 5 \end{bmatrix} \sim \begin{bmatrix} \textcircled{1} & 0 & -5 & 4 \\ 0 & \textcircled{1} & -6 & 5 \end{bmatrix}$

Corresponding system:
$$\begin{aligned} \textcircled{x_1} \quad - 5x_3 &= 4 \\ \textcircled{x_2} - 6x_3 &= 5 \end{aligned}$$

Basic variables: x_1, x_2; free variable: x_3. General solution: $\begin{cases} x_1 = 4 + 5x_3 \\ x_2 = 5 + 6x_3 \\ x_3 \text{ is free} \end{cases}$

10. $\begin{bmatrix} 1 & -2 & -1 & 3 \\ 3 & -6 & -2 & 2 \end{bmatrix} \sim \begin{bmatrix} 1 & -2 & -1 & 3 \\ 0 & 0 & 1 & -7 \end{bmatrix} \sim \begin{bmatrix} \textcircled{1} & -2 & 0 & -4 \\ 0 & 0 & \textcircled{1} & -7 \end{bmatrix}$

Corresponding system:
$$\begin{aligned} \textcircled{x_1} - 2x_2 \quad &= -4 \\ \textcircled{x_3} &= -7 \end{aligned}$$

Basic variables: x_1, x_3; free variable: x_2. General solution: $\begin{cases} x_1 = -4 + 2x_2 \\ x_2 \text{ is free} \\ x_3 = -7 \end{cases}$

11. $\begin{bmatrix} 3 & -4 & 2 & 0 \\ -9 & 12 & -6 & 0 \\ -6 & 8 & -4 & 0 \end{bmatrix} \sim \begin{bmatrix} 3 & -4 & 2 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \sim \begin{bmatrix} \textcircled{1} & -4/3 & 2/3 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

Corresponding system:
$$\begin{aligned} \textcircled{x_1} - \frac{4}{3}x_2 + \frac{2}{3}x_3 &= 0 \\ 0 &= 0 \\ 0 &= 0 \end{aligned}$$

Basic variable: x_1; free variables x_2, x_3. General solution: $\begin{cases} x_1 = \frac{4}{3}x_2 - \frac{2}{3}x_3 \\ x_2 \text{ is free} \\ x_3 \text{ is free} \end{cases}$

12. $\begin{bmatrix} 1 & -7 & 0 & 6 & 5 \\ 0 & 0 & 1 & -2 & -3 \\ -1 & 7 & -4 & 2 & 7 \end{bmatrix} \sim \begin{bmatrix} 1 & -7 & 0 & 6 & 5 \\ 0 & 0 & 1 & -2 & -3 \\ 0 & 0 & -4 & 8 & 12 \end{bmatrix} \sim \begin{bmatrix} \textcircled{1} & -7 & 0 & 6 & 5 \\ 0 & 0 & \textcircled{1} & -2 & -3 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$

Corresponding system: $\begin{aligned} \textcircled{x_1} - 7x_2 \quad + 6x_4 &= 5 \\ \textcircled{x_3} - 2x_4 &= -3 \\ 0 &= 0 \end{aligned}$

Basic variables: x_1 and x_3; free variables: x_2, x_4. General solution: $\begin{cases} x_1 = 5 + 7x_2 - 6x_4 \\ x_2 \text{ is free} \\ x_3 = -3 + 2x_4 \\ x_4 \text{ is free} \end{cases}$

13. $\begin{bmatrix} 1 & -3 & 0 & -1 & 0 & -2 \\ 0 & 1 & 0 & 0 & -4 & 1 \\ 0 & 0 & 0 & 1 & 9 & 4 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & -3 & 0 & 0 & 9 & 2 \\ 0 & 1 & 0 & 0 & -4 & 1 \\ 0 & 0 & 0 & 1 & 9 & 4 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \sim \begin{bmatrix} \textcircled{1} & 0 & 0 & 0 & -3 & 5 \\ 0 & \textcircled{1} & 0 & 0 & -4 & 1 \\ 0 & 0 & 0 & \textcircled{1} & 9 & 4 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$

Corresponding system: $\begin{aligned} \textcircled{x_1} \quad - 3x_5 &= 5 \\ \textcircled{x_2} \quad - 4x_5 &= 1 \\ \textcircled{x_4} + 9x_5 &= 4 \\ 0 &= 0 \end{aligned}$

Basic variables: x_1, x_2, x_4; free variables: x_3, x_5. General solution: $\begin{cases} x_1 = 5 + 3x_5 \\ x_2 = 1 + 4x_5 \\ x_3 \text{ is free} \\ x_4 = 4 - 9x_5 \\ x_5 \text{ is free} \end{cases}$

Note: The *Study Guide* discusses the common mistake $x_3 = 0$.

14. $\begin{bmatrix} 1 & 2 & -5 & -6 & 0 & -5 \\ 0 & 1 & -6 & -3 & 0 & 2 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \sim \begin{bmatrix} \textcircled{1} & 0 & 7 & 0 & 0 & -9 \\ 0 & \textcircled{1} & -6 & -3 & 0 & 2 \\ 0 & 0 & 0 & 0 & \textcircled{1} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$

Corresponding system:

$$\begin{aligned} \textcircled{x_1} \quad + 7x_3 \qquad\qquad &= -9 \\ \textcircled{x_2} - 6x_3 - 3x_4 \qquad &= 2 \\ \textcircled{x_5} &= 0 \\ 0 &= 0 \end{aligned}$$

Basic variables: x_1, x_2, x_5; free variables: x_3, x_4. General solution: $\begin{cases} x_1 = -9 - 7x_3 \\ x_2 = 2 + 6x_3 + 3x_4 \\ x_3 \text{ is free} \\ x_4 \text{ is free} \\ x_5 = 0 \end{cases}$

15. **a.** The system is consistent, with a unique solution.

b. The system is inconsistent. (The rightmost column of the augmented matrix is a pivot column).

16. **a.** The system is consistent, with a unique solution.

b. The system is consistent. There are many solutions because x_2 is a free variable.

17. $\begin{bmatrix} 2 & 3 & h \\ 4 & 6 & 7 \end{bmatrix} \sim \begin{bmatrix} \textcircled{2} & 3 & h \\ 0 & 0 & 7-2h \end{bmatrix}$ The system has a solution only if $7 - 2h = 0$, that is, if $h = 7/2$.

18. $\begin{bmatrix} 1 & -3 & -2 \\ 5 & h & -7 \end{bmatrix} \sim \begin{bmatrix} \textcircled{1} & -3 & -2 \\ 0 & h+15 & 3 \end{bmatrix}$ If $h + 15$ is zero, that is, if $h = -15$, then the system has no solution, because 0 cannot equal 3. Otherwise, when $h \neq -15$, the system has a solution.

19. $\begin{bmatrix} 1 & h & 2 \\ 4 & 8 & k \end{bmatrix} \sim \begin{bmatrix} \textcircled{1} & h & 2 \\ 0 & 8-4h & k-8 \end{bmatrix}$

a. When $h = 2$ and $k \neq 8$, the augmented column is a pivot column, and the system is inconsistent.

b. When $h \neq 2$, the system is consistent and has a unique solution. There are no free variables.

c. When $h = 2$ and $k = 8$, the system is consistent and has many solutions.

20. $\begin{bmatrix} 1 & 3 & 2 \\ 3 & h & k \end{bmatrix} \sim \begin{bmatrix} \textcircled{1} & 3 & 2 \\ 0 & h-9 & k-6 \end{bmatrix}$

a. When $h = 9$ and $k \neq 6$, the system is inconsistent, because the augmented column is a pivot column.

b. When $h \neq 9$, the system is consistent and has a unique solution. There are no free variables.

c. When $h = 9$ and $k = 6$, the system is consistent and has many solutions.

21. **a.** False. See Theorem 1.

b. False. See the second paragraph of the section.

c. True. Basic variables are defined after equation (4).

d. True. This statement is at the beginning of *Parametric Descriptions of Solution Sets*.

e. False. The row shown corresponds to the equation $5x_4 = 0$, which does not by itself lead to a contradiction. So the system might be consistent or it might be inconsistent.

22. **a.** False. See the statement preceding Theorem 1. Only the *reduced* echelon form is unique.

b. False. See the beginning of the subsection *Pivot Positions*. The pivot positions in a matrix are determined completely by the positions of the leading entries in the nonzero rows of any echelon form obtained from the matrix.

c. True. See the paragraph after Example 3.

d. False. The existence of at least one solution is not related to the presence or absence of free variables. If the system is inconsistent, the solution set is empty. See the solution of Practice Problem 2.

e. True. See the paragraph just before Example 4.

23. Yes. The system is consistent because with three pivots, there must be a pivot in the third (bottom) row of the coefficient matrix. The reduced echelon form cannot contain a row of the form [0 0 0 0 0 1].

24. The system is inconsistent because the pivot in column 5 means that there is a row of the form [0 0 0 0 1]. Since the matrix is the *augmented* matrix for a system, Theorem 2 shows that the system has no solution.

25. If the coefficient matrix has a pivot position in every row, then there is a pivot position in the bottom row, and there is no room for a pivot in the augmented column. So, the system is consistent, by Theorem 2.

26. Since there are three pivots (one in each row), the augmented matrix must reduce to the form

$$\begin{bmatrix} \textcircled{1} & 0 & 0 & a \\ 0 & \textcircled{1} & 0 & b \\ 0 & 0 & \textcircled{1} & c \end{bmatrix} \quad \text{and so} \quad \begin{aligned} \textcircled{x_1} \qquad\qquad &= a \\ \textcircled{x_2} \qquad &= b \\ \textcircled{x_3} &= c \end{aligned}$$

No matter what the values of a, b, and c, the solution exists and is unique.

27. *"If a linear system is consistent, then the solution is unique if and only if every column in the coefficient matrix is a pivot column; otherwise there are infinitely many solutions."*

This statement is true because the free variables correspond to *nonpivot* columns of the coefficient matrix. The columns are all pivot columns if and only if there are no free variables. And there are no free variables if and only if the solution is unique, by Theorem 2.

28. Every column in the augmented matrix *except the rightmost column* is a pivot column, and the rightmost column is *not* a pivot column.

29. An underdetermined system always has more variables than equations. There cannot be more basic variables than there are equations, so there must be at least one free variable. Such a variable may be assigned infinitely many different values. If the system is consistent, each different value of a free variable will produce a different solution.

30. Example:
$$\begin{aligned} x_1 + x_2 + x_3 &= 4 \\ 2x_1 + 2x_2 + 2x_3 &= 5 \end{aligned}$$

31. Yes, a system of linear equations with more equations than unknowns can be consistent.

Example (in which $x_1 = x_2 = 1$):
$$\begin{aligned} x_1 + x_2 &= 2 \\ x_1 - x_2 &= 0 \\ 3x_1 + 2x_2 &= 5 \end{aligned}$$

32. According to the numerical note in Section 1.2, when $n = 30$ the reduction to echelon form takes about $2(30)^3/3 = 18{,}000$ flops, while further reduction to reduced echelon form needs at most $(30)^2 = 900$ flops. Of the total flops, the "backward phase" is about $900/18900 = .048$ or about 5%.

When $n = 300$, the estimates are $2(300)^3/3 = 18{,}000{,}000$ phase for the reduction to echelon form and $(300)^2 = 90{,}000$ flops for the backward phase. The fraction associated with the backward phase is about $(9\times10^4)/(18\times10^6) = .005$, or about .5%.

33. For a quadratic polynomial $p(t) = a_0 + a_1t + a_2t^2$ to exactly fit the data (1, 12), (2, 15), and (3, 16), the coefficients a_0, a_1, a_2 must satisfy the systems of equations given in the text. Row reduce the augmented matrix:

$$\begin{bmatrix} 1 & 1 & 1 & 12 \\ 1 & 2 & 4 & 15 \\ 1 & 3 & 9 & 16 \end{bmatrix} \sim \begin{bmatrix} 1 & 1 & 1 & 12 \\ 0 & 1 & 3 & 3 \\ 0 & 2 & 8 & 4 \end{bmatrix} \sim \begin{bmatrix} 1 & 1 & 1 & 12 \\ 0 & 1 & 3 & 3 \\ 0 & 0 & 2 & -2 \end{bmatrix} \sim \begin{bmatrix} 1 & 1 & 1 & 12 \\ 0 & 1 & 3 & 3 \\ 0 & 0 & 1 & -1 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & 1 & 0 & 13 \\ 0 & 1 & 0 & 6 \\ 0 & 0 & 1 & -1 \end{bmatrix} \sim \begin{bmatrix} \textcircled{1} & 0 & 0 & 7 \\ 0 & \textcircled{1} & 0 & 6 \\ 0 & 0 & \textcircled{1} & -1 \end{bmatrix}$$

The polynomial is $p(t) = 7 + 6t - t^2$.

34. **[M]** The system of equations to be solved is:

$$\begin{aligned}
a_0 + a_1\cdot 0 + a_2\cdot 0^2 + a_3\cdot 0^3 + a_4\cdot 0^4 + a_5\cdot 0^5 &= 0\\
a_0 + a_1\cdot 2 + a_2\cdot 2^2 + a_3\cdot 2^3 + a_4\cdot 2^4 + a_5\cdot 2^5 &= 2.90\\
a_0 + a_1\cdot 4 + a_2\cdot 4^2 + a_3\cdot 4^3 + a_4\cdot 4^4 + a_5\cdot 4^5 &= 14.8\\
a_0 + a_1\cdot 6 + a_2\cdot 6^2 + a_3\cdot 6^3 + a_4\cdot 6^4 + a_5\cdot 6^5 &= 39.6\\
a_0 + a_1\cdot 8 + a_2\cdot 8^2 + a_3\cdot 8^3 + a_4\cdot 8^4 + a_5\cdot 8^5 &= 74.3\\
a_0 + a_1\cdot 10 + a_2\cdot 10^2 + a_3\cdot 10^3 + a_4\cdot 10^4 + a_5\cdot 10^5 &= 119
\end{aligned}$$

The unknowns are $a_0, a_1, \ldots, a_5$. Use technology to compute the reduced echelon of the augmented matrix:

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 2 & 4 & 8 & 16 & 32 & 2.9 \\ 1 & 4 & 16 & 64 & 256 & 1024 & 14.8 \\ 1 & 6 & 36 & 216 & 1296 & 7776 & 39.6 \\ 1 & 8 & 64 & 512 & 4096 & 32768 & 74.3 \\ 1 & 10 & 10^2 & 10^3 & 10^4 & 10^5 & 119 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 2 & 4 & 8 & 16 & 32 & 2.9 \\ 0 & 0 & 8 & 48 & 224 & 960 & 9 \\ 0 & 0 & 24 & 192 & 1248 & 7680 & 30.9 \\ 0 & 0 & 48 & 480 & 4032 & 32640 & 62.7 \\ 0 & 0 & 80 & 960 & 9920 & 99840 & 104.5 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 2 & 4 & 8 & 16 & 32 & 2.9 \\ 0 & 0 & 8 & 48 & 224 & 960 & 9 \\ 0 & 0 & 0 & 48 & 576 & 4800 & 3.9 \\ 0 & 0 & 0 & 192 & 2688 & 26880 & 8.7 \\ 0 & 0 & 0 & 480 & 7680 & 90240 & 14.5 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 2 & 4 & 8 & 16 & 32 & 2.9 \\ 0 & 0 & 8 & 48 & 224 & 960 & 9 \\ 0 & 0 & 0 & 48 & 576 & 4800 & 3.9 \\ 0 & 0 & 0 & 0 & 384 & 7680 & -6.9 \\ 0 & 0 & 0 & 0 & 1920 & 42240 & -24.5 \end{bmatrix}$$

$$\sim\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 2 & 4 & 8 & 16 & 32 & 2.9 \\ 0 & 0 & 8 & 48 & 224 & 960 & 9 \\ 0 & 0 & 0 & 48 & 576 & 4800 & 3.9 \\ 0 & 0 & 0 & 0 & 384 & 7680 & -6.9 \\ 0 & 0 & 0 & 0 & 0 & 3840 & 10 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 2 & 4 & 8 & 16 & 32 & 2.9 \\ 0 & 0 & 8 & 48 & 224 & 960 & 9 \\ 0 & 0 & 0 & 48 & 576 & 4800 & 3.9 \\ 0 & 0 & 0 & 0 & 384 & 7680 & -6.9 \\ 0 & 0 & 0 & 0 & 0 & 1 & .0026 \end{bmatrix}$$

$$\sim\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 2 & 4 & 8 & 16 & 0 & 2.8167 \\ 0 & 0 & 8 & 48 & 224 & 0 & 6.5000 \\ 0 & 0 & 0 & 48 & 576 & 0 & -8.6000 \\ 0 & 0 & 0 & 0 & 384 & 0 & -26.900 \\ 0 & 0 & 0 & 0 & 0 & 1 & .002604 \end{bmatrix} \sim\cdots\sim \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 1.7125 \\ 0 & 0 & 1 & 0 & 0 & 0 & -1.1948 \\ 0 & 0 & 0 & 1 & 0 & 0 & .6615 \\ 0 & 0 & 0 & 0 & 1 & 0 & -.0701 \\ 0 & 0 & 0 & 0 & 0 & 1 & .0026 \end{bmatrix}$$

Thus $p(t) = 1.7125t - 1.1948t^2 + .6615t^3 - .0701t^4 + .0026t^5$, and $p(7.5) = 64.6$ hundred lb.

Notes: In Exercise 34, if the coefficients are retained to higher accuracy than shown here, then $p(7.5) = 64.8$. If a polynomial of lower degree is used, the resulting system of equations is overdetermined. The augmented matrix for such a system is the same as the one used to find p, except that at least column 6 is missing. When the augmented matrix is row reduced, the sixth row of the augmented matrix will be entirely zero except for a nonzero entry in the augmented column, indicating that no solution exists.

Exercise 34 requires 25 row operations. It should give students an appreciation for higher-level commands such as `gauss` and `bgauss`, discussed in Section 1.4 of the *Study Guide*. The command `ref` (reduced echelon form) is available, but I recommend postponing that command until Chapter 2.

The *Study Guide* includes a "Mathematical Note" about the phrase, "If and only if," used in Theorem 2.