Chapter 4.5 Complements and Conditional Probability

Complements

\[ P(\overline{A}) = 1 - P(A) \]

Use this for the probability of at least one.

**Ex.** You make a guess at 7 multiple-choice questions each with 5 choices. Find the probability of getting at least 1 correct.

\[
P(\text{at least 1 correct}) = P(\text{1 or more correct}) \\
= P(\text{1 correct}) + P(\text{2 correct}) + \ldots + P(\text{7 correct})
\]

Too much work!

\[
P(\text{at least 1 correct}) = 1 - P(\text{0 correct}) = 1 - \left(\frac{4}{5}\right)^7 = 0.7903
\]

Conditional Probability

\[
P(B|A) = \frac{P(A \text{ and } B)}{P(A)}
\]

The probability of \(B\) given \(A\) has occurred

(You don’t need to use this formula.)

**Ex. Titanic Data**

<table>
<thead>
<tr>
<th></th>
<th>Men</th>
<th>Women</th>
<th>Boys</th>
<th>Girls</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Survived</td>
<td>332</td>
<td>318</td>
<td>29</td>
<td>27</td>
<td>706</td>
</tr>
<tr>
<td>Died</td>
<td>1360</td>
<td>104</td>
<td>35</td>
<td>18</td>
<td>1517</td>
</tr>
<tr>
<td></td>
<td>1692</td>
<td>422</td>
<td>64</td>
<td>45</td>
<td>2223</td>
</tr>
</tbody>
</table>

a. Find the probability of selecting someone who is a man.

\[ P(M) = \frac{1692}{2223} = 0.7611 \]

b. Find the probability of selecting someone who is a man given that he died.

\[ P(M|D) = \frac{1360}{1517} = 0.8965 \]

c. Find the probability of selecting someone who survived given that it is a man.

\[ P(D|M) = \frac{332}{1692} = 0.1962 \]

d. Find the probability of selecting someone who is a man or boy given that he survived.

\[ P(M \text{ or } B|S) = \frac{332 + 29}{706} = 0.5133 \]

e. Find the probability of choosing two people and they both survive.

\[ P(SS) = \frac{706}{2223} \cdot \frac{705}{2222} = 0.1008 \]