Chapter 9.2 Inferences About Two Proportions

Hypothesis Tests and Confidence Intervals comparing 2 proportions

Assumptions:
- Both samples are independent
- \( np \geq 5 \) and \( nq \geq 5 \)

Test Statistic*:
\[
z = \frac{(\hat{p}_1 - \hat{p}_2) - (p_1 - p_2)}{\sqrt{\frac{pq}{n_1} + \frac{pq}{n_2}}} \quad \bar{p} = \frac{x_1 + x_2}{n_1 + n_2} \quad \bar{q} = 1 - \bar{p}
\]

Confidence Interval*:
\[
E = Z_{\alpha/2} \sqrt{\frac{\hat{p}_1 \hat{q}_1}{n_1} + \frac{\hat{p}_2 \hat{q}_2}{n_2}}
\]

*You can use the calculator function instead of the formula.

Example:
The Santa Clarita County Clerk wishes to improve voter registration. One method under consideration is to send reminders in the mail to all citizens in the county who are eligible to register. As a pilot study to determine if this method will improve voter registration, a random sample of 1100 potential voters was taken. Then this sample was divided into two groups.

Group 1: 500 potential voters; no registration reminders sent; 248 registered to vote
Group 2: 600 potential voters; registration reminders sent; 332 registered to vote

Do these data support the claim that the proportion of voters who register was greater in the group that received reminders than in the group that did not? Use a 1% level of significance.

(See the next page for how to find the test statistic and P–value using the calculator.)

Claim: \( p_1 < p_2 \)

\[
H_0: p_1 = p_2 \\
H_1: p_1 < p_2
\]

\[
x_1 = 248 \quad x_2 = 332 \\
n_1 = 500 \quad n_2 = 600
\]

\( \alpha = 0.01 \)

TS: \( z = -1.8965 \)

\[P–value = P(z < -1.8965) = 0.0289 > 0.01\]

Do Not Reject \( H_0 \)

There is not sufficient evidence to support the claim that \( p_1 < p_2 \)

It appears that sending reminders does not increase the proportion of registered voters.
For the informal conclusion on a 2 tail–test where you Reject $H_0$ be sure to say which population parameter is greater. Do not leave in the form “It appears there is a difference” since this does not give useful information.

**Note:** We will be using the calculator function to compute the Test Statistic and Confidence Interval not the formulas.

**Finding the Test Statistic on the TI–83/4**

STAT TESTS 2–PropZTest

Enter the data
$p_1$: refers to the alternate hypothesis, make note of the direction of the inequality symbol.
Then Calculate

The Test Statistic is $z$ and the $P$–value is $p$

For this example:
TS: $z = -1.8965$
$P$–value $= P(z < -1.8965) = 0.0289$

Note: if you are given $n$ and $\hat{p}$ then use the formula $x = n\hat{p}$ to find $x$. 
Confidence Intervals:

The objective in constructing a Confidence Interval is to determine if it contains zero. If it does contain zero, then there is not a significant difference between the proportions which suggest they are not different. If the Confidence Interval does not contain zero, then it appears the proportions are different.

When constructing a Confidence Interval that corresponds to a 1 tail hypothesis at a significance level of $\alpha$ you need to double $\alpha$ for the Confidence Interval.

Finding a Confidence Interval on the TI–83/4

The CI is $(-.1276, .0129)$
Write in the form: $-.1276 < p_1 - p_2 < .0129$ (round to 4 decimal places)

Note that this interval does contain zero so the conclusion is that there is no significant difference between the population proportions. In other words, the reminders do not seem to increase the proportion of voter registration.