Chapter 9.4 Inferences from Matched Pairs

Two samples are dependent if there is a relation between the specific values of the distributions. The data will come in pairs.

For the Independent cases the test is to compare the difference of the means whereas the Dependent (Matched Pairs) case the test is to compare the mean of the differences.

Examples:
- Weights of people before starting a diet and 6 months later
- Pulses of runners just after running a mile and 15 minutes later

Requirements:
- Matched Pairs Samples
- Simple Random Samples
- Each Population is Normally Distributed or \( n > 30 \)

Notation:
- \( d \) = differences of the paired data
- \( \mu_d \) = mean of the differences of the population paired data
- \( \bar{d} \) = mean of the differences of the sample paired data
- \( s_d \) = standard deviation of the differences of the sample paired data
- \( n \) = number of paired data

Test Statistic: 
\[
    t = \frac{\bar{d} - \mu_d}{s_d / \sqrt{n}}
\]
\( df = n - 1 \) (state what \( d \) is)

Confidence Interval: 
\[
    \bar{d} - E < \mu_d < \bar{d} + E
\]
\[
    E = t_{\alpha/2} \frac{s_d}{\sqrt{n}}
\]

Compute the Test Statistic and Confidence Interval using these formulas not the calculator function.
Computing the mean and standard deviation on the TI

Enter the data for the variables in L₁ and L₂
Move the cursor to L₃ and type L₁ – L₂ then Enter

L₃ contains the differences of the variables

\[ \bar{d} = \bar{x} \text{ and } s_d = S_x \]
\[ \mu_d = 0 \text{ (always)} \]
Use this information to find the Test Statistic (or Confidence Interval) using the formula.
Example:

Five students took a math test before and after tutoring. Their scores were as follows.

<table>
<thead>
<tr>
<th>Subject</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
</tr>
</thead>
<tbody>
<tr>
<td>Before</td>
<td>72</td>
<td>66</td>
<td>70</td>
<td>65</td>
<td>72</td>
</tr>
<tr>
<td>After</td>
<td>80</td>
<td>75</td>
<td>68</td>
<td>76</td>
<td>84</td>
</tr>
</tbody>
</table>

a. Using a 0.05 level of significance, test the claim that the tutoring is effective in raising math scores.

Claim: \( \mu_d < 0 \) (d = Before – After)

\( H_0: \mu_d = 0 \)

\( H_1: \mu_d < 0 \)

\[ \bar{d} = -7.6 \]

\[ s_d = 5.5946 \]

\( n = 5 \)

\( df = 4 \)

\( \alpha = 0.05 \)

\[ t_{TS} = \frac{-7.6 - 0}{5.5946} = -3.0376 \]

(Don’t worry about the P-value)

Reject \( H_0 \)

The sample data support the claim that \( \mu_d < 0 \)

It appears that tutoring is effective in raising math scores.

b. Construct a 90% confidence interval and interpret the results.

\( \alpha = 0.10 \)

\[ t_{\alpha/2} = 2.132 \]

\[ E = \frac{2.132 \times 5.5946}{\sqrt{5}} = 5.3342 \]

\[ \bar{d} - E < \mu_d < \bar{d} + E \]

\[ -7.6 - 5.3342 < \mu_d < -7.6 + 5.3342 \]

\[ -12.9342 < \mu_d < -2.2658 \]

Since the confidence interval does not include zero and all the values are negative it appears the After scores are larger than the Before scores and tutoring is effective in raising math scores.