Comparing Two Independent Population Means with $\sigma$ unknown

Two samples are independent if there is no relation between the specific values of the two distributions.

Example 1:
- Comparing the difference in average exam scores of men and women
- Comparing the difference in the rate of seat belt use between different age groups

Requirements:
- Independent Samples
- Simple Random Samples
- Either both populations are normal or both sample sizes are greater than 30

Properties:
$\bar{x}_1 - \bar{x}_2$ = the difference of the sample means
The distribution is $\bar{x}_1 - \bar{x}_2 \sim t_{df}$ df from the calculator

The Null and Alternate Hypothesis is in one of these forms. (Different than the text)

<table>
<thead>
<tr>
<th>$H_0$: $\mu_1 = \mu_2$</th>
<th>$H_0$: $\mu_1 = \mu_2$</th>
<th>$H_0$: $\mu_1 = \mu_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$H_a$: $\mu_1 \neq \mu_2$</td>
<td>$H_a$: $\mu_1 &lt; \mu_2$</td>
<td>$H_a$: $\mu_1 &gt; \mu_2$</td>
</tr>
</tbody>
</table>

Follow the same procedure from chapter 9 for hypothesis tests.
Example 2:
A consumer group is testing camp stoves. To test the heating capacity, they measure the time required to bring 2 quarts of water from 50°F to boiling. Two competing models are under consideration. Twenty-four stoves of each model were tested and the following results were obtained.

Model 1: $\bar{x}_1 = 11.4$ min, $n_1 = 24$, $s_1 = 2.5$ min
Model 2: $\bar{x}_2 = 9.9$ min, $n_2 = 24$, $s_2 = 3.0$ min

Use a hypotheses test to determine if there is a difference between the performances of these two models? Use a 5% significance level.

$H_0$: $\mu_1 = \mu_2$

$H_a$: $\mu_1 \neq \mu_2$

$\bar{x}_1 - \bar{x}_2 = 11.4 - 9.9 = 1.5$

$\alpha = 0.05$

$t_{44.5513} = 1.8818$

$P-value = 2P( t > 1.8818) = 0.0664 > 0.05$

Do Not Reject $H_0$

There is not sufficient evidence to conclude that there is a difference in the heating times of these models.

Calculator Instructions

From the Stat mode press [F3], [F2], [F2], [F2] (Variable)

Enter the data

Pooled: Off

Execute
Example 3

A recent article in the school newspaper claims that male college students spend more money on coffee than female college students. A random sample of college coffee drinkers were asked how much money they spent on coffee in the last month. The results are displayed below. Use the data to test the newspaper’s claim at a 5% significance level.

<table>
<thead>
<tr>
<th>Male</th>
<th>73</th>
<th>63</th>
<th>88</th>
<th>55</th>
<th>81</th>
<th>90</th>
<th>44</th>
<th>52</th>
<th>97</th>
</tr>
</thead>
<tbody>
<tr>
<td>Female</td>
<td>46</td>
<td>52</td>
<td>75</td>
<td>51</td>
<td>44</td>
<td>28</td>
<td>93</td>
<td>45</td>
<td>66</td>
</tr>
</tbody>
</table>

\[ H_0: \mu_M = \mu_F \]
\[ H_1: \mu_M > \mu_F \]
\[ \alpha = 0.05 \]
\[ df = 16.7051 \]

\[ \bar{X}_M - \bar{X}_F = 71.4444 - 54.2727 = 17.1717 \]
\[ \bar{X}_1 - \bar{X}_2 \sim t_{16.7051} \]

TS: \( t = 2.0879 \)

\[ P-value = P(t > 2.0879) = 0.0262 < 0.05 \]

Reject \( H_0 \)

There is sufficient evidence to conclude that male college students spend more money on coffee than female college students.

Calculator Instructions

Enter the data in List 1 and List 2

[F3], [F2], [F2]

[F1] (List) Make sure the List match where your data is.

Pooled: Off

Execute
Comparing Two Independent Population Proportions

Assumptions:
- Both samples are independent
- np ≥ 5 and nq ≥ 5

Properties:
- $P'_A - P'_B$ = the difference of the sample proportions
- The distribution is $P'_A - P'_B \sim N\left(0, \frac{|P'_A - P'_B|}{z\text{-score}}\right)$

Note that this is not the form of the distribution given in the text but since the z-score is obtained from the calculator you can find the distribution with this formula.

The Null and Alternate Hypothesis is in one of these forms.

\[
\begin{array}{ccc}
H_0: p_A = p_B & H_0: p_A = p_B & H_0: p_A = p_B \\
H_A: p_A \neq p_B & H_A: p_A < p_B & H_A: p_A > p_B \\
\end{array}
\]

See the last page for P-value interpretation examples
**Example 4:**
The Alameda county Clerk wishes to improve voter registration. One method under consideration is to send reminders in the mail to all citizens in the county who are eligible to register. As a pilot study to determine if this method will improve voter registration, a random sample of potential voters was taken and divided into two groups.

- **Group 1:** 475 potential voters; no registration reminders sent; 49% registered to vote
- **Group 2:** 538 potential voters; registration reminders sent; 54% registered to vote

Based on the data is there evidence that the reminders increase the rate of voter registration? Use 1% level of significance.

**Null Hypothesis (H₀):** \( p_1 = p_2 \)

**Alternative Hypothesis (H₁):** \( p_1 < p_2 \)

- \( p'_1 = 0.49 \)
- \( p'_2 = 0.54 \)
- \( n_1 = 475 \)
- \( n_2 = 538 \)
- \( x_1 = 233 \) (rounded)
- \( x_2 = 291 \) (rounded)
- \( \alpha = 0.01 \)

\[
\begin{align*}
P'_1 - P'_2 &= 0.49 - 0.54 = -0.05 \\
P'_1 - P'_2 &\sim N\left[0, \frac{0.05}{1.6009}\right]
\end{align*}
\]

**Test Statistic (TS):** \( z = -1.6009 \)

**P-value:** \( P(z < -1.6009) = 0.0547 > 0.01 \)

Do Not Reject \( H_0 \)

There is not sufficient evidence to show that the reminders increase the rate of voter registration.

**Calculator Instructions**

From the Stat mode press \([F3],[F1],[F3]\), enter the data, highlight Execute then press \([F4]\).

Enter the data

(If the \( x \) values must be whole numbers)

Execute

```
<table>
<thead>
<tr>
<th>Z-Test for Two Proportions</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \hat{p}_1 = 0.49 )</td>
</tr>
<tr>
<td>( n_1 = 475 )</td>
</tr>
<tr>
<td>( x_1 = 233 )</td>
</tr>
<tr>
<td>( \hat{p}'_1 )</td>
</tr>
<tr>
<td>( z = -1.6009 )</td>
</tr>
<tr>
<td>( p-value = 0.0547 )</td>
</tr>
</tbody>
</table>
```
Matched Pair Samples

Two samples are dependent if there is a relation between the specific values of the distributions. The data will come in pairs.

For the Independent cases the test is to compare the difference of the means whereas the Dependent (Matched Pairs) case the test is to compare the mean of the differences.

**Example 5:**
- Weights of people before starting a diet and 6 months later
- Pulses of runners just after running a mile and 15 minutes later

**Requirements:**
- Matched Pairs Samples
- Simple Random Samples
- Each Population is Normally Distributed or n > 30

**Notation:**
- \( d \) = differences of the paired data
- \( \mu_d \) = mean of the differences of the population paired data
- \( \bar{X}_d \) = mean of the differences of the sample paired data
- \( s_d \) = standard deviation of the differences of the sample paired data
- \( n \) = number of paired data

**Test Statistic:**
\[
t = \frac{\bar{X}_d - \mu_d}{s_d \sqrt{n}}
\]

**Distribution:**
\( \bar{X}_d \sim t_{df} \quad df = n-1 \)

\( \bar{X}_d \) is the mean of the differences

The Null and Alternate Hypothesis is in one of these forms.

<table>
<thead>
<tr>
<th>( H_0: \mu_d = 0 )</th>
<th>( H_0: \mu_d = 0 )</th>
<th>( H_0: \mu_d = 0 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( H_a: \mu_d \neq 0 )</td>
<td>( H_a: \mu_d &lt; 0 )</td>
<td>( H_a: \mu_d &gt; 0 )</td>
</tr>
</tbody>
</table>

State what \( d \) represents
Example 6:
Five students took a math test before and after tutoring. Their scores were as follows.

<table>
<thead>
<tr>
<th>Subject</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
</tr>
</thead>
<tbody>
<tr>
<td>Before</td>
<td>72</td>
<td>66</td>
<td>70</td>
<td>65</td>
<td>72</td>
</tr>
<tr>
<td>After</td>
<td>80</td>
<td>75</td>
<td>68</td>
<td>76</td>
<td>84</td>
</tr>
</tbody>
</table>

Using a 0.05 level of significance, test the claim that the tutoring is effective in raising math scores.

This must be included

\[ \overline{d} = \text{Before} - \text{After} \]

\[ H_0: \mu_d = 0 \]

\[ H_1: \mu_d < 0 \]

\( n = 5 \)

\( df = 4 \)

\( \alpha = 0.05 \)

\[ \overline{X}_d \sim t_d \]

\[ \overline{X}_d = -7.6 \]

\[ TS: t = -3.0376 \]

\[ P\text{-value} = 0.0192 < 0.05 \]

Reject \( H_0 \)

There is sufficient evidence to conclude that tutoring is effective in raising math scores.

Calculator Instructions

Enter the data in List 1 and List 2

Move the cursor to List 3 and type

SHIFT List 1 – SHIFT List 2   EXE

List 3 contains the differences of the variables

[F3], [F2], [F1] Set the List to 3 since that is where the differences are.

EXE
P–value Interpretation

Example 7:
Write the interpretation for each of the p–values from the above examples.

From Example 2
The P–value of 0.0664 is the probability that the difference in sample mean heating times of Model 1 and Model 2 camp stoves is 1.5 minutes or more if the null hypothesis is true and there is actually no difference in the heating times.

From Example 3
The P–value of 0.0262 is the probability that the difference in the mean spending on coffee of males and females is at least $17.17 if the null hypothesis is true and there is no difference in the amount of money spend on coffee between males and females.

From Example 4
The P–value of 0.0547 is the probability that the difference in the proportion of registered voters who do not get the reminders and do get the reminders is –0.05 or less if the null hypothesis is true and there is no difference in the proportion of registered voters between the group that get the reminders and do not get the reminders.

From Example 6
The P–value of 0.0192 is the probability that the mean of the differences between the test scores before and after tutoring is -7.6 or less if the null hypothesis is true and there is no difference in test scores after tutoring.
Example 8:
As a part of the United States Department of Agriculture's Super Dump cleanup efforts in
the early 1900's, various sites in the country were targeted for cleanup. Some of the
rivers near the targeted sites may be contaminated with pesticides. Measurements of the
concentration of aldrin (a commonly used pesticide) in parts per million (ppm) were
taken at randomly selected locations near dump sites (Y) and random sites 100 miles
away from the dump sites (Z). Below is the summary statistics from the tests:

<table>
<thead>
<tr>
<th>Rivers</th>
<th>n</th>
<th>Min.</th>
<th>Q1</th>
<th>Median</th>
<th>Q3</th>
<th>Max</th>
</tr>
</thead>
<tbody>
<tr>
<td>Y (near)</td>
<td>20</td>
<td>3.2</td>
<td>4.1</td>
<td>5</td>
<td>6.1</td>
<td>6.5</td>
</tr>
<tr>
<td>Z (far)</td>
<td>20</td>
<td>3</td>
<td>3.5</td>
<td>4.3</td>
<td>4.6</td>
<td>5.1</td>
</tr>
</tbody>
</table>

a) Draw side-by-side box plots that compare the two competitors.

b) Compare the two distributions. Include the values of the medians, quarterlies,
and the IQR. What is similar and what is different between the two groups of
rivers?

c) In a test of significance, the null and alternative hypothesis is

$H_0: \mu_{near} = \mu_{far}$ vs $H_a: \mu_{near} > \mu_{far}$

The p-value of this test is 0.0928. Explain what this number means in context of
the problem.

d) Based on the p-value, what conclusion can be drawn in the context of this study?
Use a significance level of 0.05 and include justification.

e) Based on your conclusion from part (d), which type of error, Type I or Type II,
could have been made? What is one potential consequence of this error?

Answers:

a. [Diagram]

b. The minimum values are almost the same and the values of the median, lower
and upper quartiles from River Y are all greater than River Z. The IQR from
River Y is greater than River Z (2 ppm vs 1.1 ppm) indicating more variation in
aldrin concentration levels. The range is greater in the River Y. It appears there
is more aldrin concentration in River Y.

c. The P-value of 0.0928 is the probability of observing a difference between the
two sample means as large or larger than the one observed, if the concentration
of aldrin from the rivers are the same.

d. Because the p-value of 0.0928 is greater than 0.05, the null hypotheses should
not be rejected. There is not sufficient evidence to conclude the rivers close to
the cleanup have a higher concentration of aldrin than rivers 100 miles away.

e. Because the null hypothesis was not rejected a Type II error could have
occurred. A possible consequence is that the rivers do have a high concentration
of aldrin and health issues could occur such as adverse side effects from
drinking the river water.