Chapter 6 The Normal Distribution (TI)

The **Standard Normal Distribution** is a normal distribution (bell-shaped) with \( \mu = 0 \) and \( \sigma = 1 \).

A **Non-Standard Normal Distribution** is a normal distribution (bell-shaped) with \( \mu \neq 0 \) or \( \sigma \neq 1 \) (or both).

**Basic Properties:**
- The total area under the curve and above the horizontal axis is 1
- The curve extends indefinitely in both directions approaching, but never touching, the horizontal axis.
- The curve is symmetric about 0; that is, the part of the curve to the left of 0 is the mirror image of the part of the curve to the right.
- Almost all the area under the curve lies between –3 and 3.
- \( X \sim N(\mu, \sigma) \)

Recall: A \( z \)-score is the number of standard deviation a data point is from the mean.

\[
z = \frac{x - \mu}{\sigma}
\]

**Example 1:**
For \( X \sim N(80, 7) \) find the \( z \)-score for:

a. \( x = 91 \)
\[
z = \frac{91 - 80}{7} = 1.5714
\]

b. \( x = 65 \)
\[
z = \frac{65 - 80}{7} = -2.1429
\]

**Finding Probabilities**
1. Write the function (in \( x \))
2. Draw the \( x \)-curve and shade the region
3. Convert to a \( z \)-score (round to 4 decimal places)
4. Write the function (in \( z \))
5. Draw the \( z \)-curve and shade the region
6. Use the calculator to find the probability (round to 4 decimal places) include the calculator function

**Calculator:** (See below for more details)

**Finding probabilities**
- \( \text{Normalcdf}(\text{lowerbound}, \text{upperbound}, 0, 1) \)
  - if \( P(X < x) \) then \( \text{lowerbound} \) is \(-1E99\)
  - if \( P(X > x) \) then \( \text{upperbound} \) is \(1E99\)

**Finding scores**
- \( \text{invNorm}(\text{percentile}, \mu, \sigma) \)
Example 2:
Quick Start Company makes 12–volt batteries. After many years of testing, the company knows that the average life of their battery is normally distributed with a mean of 45 months and a standard deviation of 8 months.

a. What is the probability that a randomly selected battery will last less than 36 months?

\[ X \sim N(45, 8) \]

\[ P(x < 36) \]

\[ z = \frac{36 - 45}{8} = -1.125 \]

\[ P(z < -1.125) = \text{Normalcdf}(-E99, -1.125, 0, 1) = 0.1303 \]

(see below for the calculator commands)

b. If Quick Start guarantees a full refund on any battery that fails within the 36–month period after purchase, what percentage of its batteries will the company expect to replace?

They will replace about 13% of the batteries.

Turn on STAT WIZARD by pressing [MODE] then set STAT WIZARD to ON

If you calculator does not have STAT WIZARD then start at the next step and you will see the last 2 screen shots

Press [2nd] [DIST] (next to CLEAR) then normalcdf [ENTER]

lower : (-)1 [2nd][EE] (above 7) 99
upper: -1.125
\( \mu : 0 \)
\( \sigma : 1 \)

[ENTER]

\[ \text{normalcdf}(-1E99, -1.125, 0, 1) \]

[ENTER]

\[ \text{normalcdf}(-1E99, -1.125, 0, 1) = 0.1302945643 \]
Finding a score given a probability or percentile:

1. Write the function (in x)
2. Draw the x–curve and shade the region
3. Use the calculator to find the score (round to 4 decimal places) include the calculator function

Example 3:
If Quick Start does not want to replace more than 10% of its batteries for how long should the company guarantee the batteries?

\[ X \sim N(45, 8) \]

\[ P(x < c) = 0.10, \text{ find } c \]

\[ c = \text{invNorm}(0.10, 45, 8) = 34.7476 \]

They should guarantee the batteries for 35 months.

Press [2^{nd}] [DIST] (next to CLEAR) then invNormalcdf [ENTER]

| Press [2^{nd}] [DIST] (next to CLEAR) then invNormalcdf [ENTER] | \begin{array} {|c|c|}
| \text{area: .10} & \text{invNorm} \\
| \mu: 45 & \text{area: .10} \\
| \sigma: 8 & \mu: 45 \\
| & \sigma: 8 \\
| & \text{Paste} \\
| [ENTER] & 34.74758747 \\
| [ENTER] & \end{array} |
Example 4: 
X~N(80, 5)

a. \( P(x > 87) \)

\[
z = \frac{87 - 80}{5} = 1.4
\]

\[P(z > 1.4) = \text{Normalcdf}(1.4, 99, 0, 1) = 0.0808\]

b. \( P(72 < x < 87) \)

\[
z = \frac{72 - 80}{5} = -1.6
\]

\[
z = \frac{87 - 80}{5} = 1.4
\]

\[P(-1.6 < z < 1.4) = \text{Normalcdf}(-1.6, 1.4, 0, 1) = 0.8644\]

c. Find \( P_{95} \)

\[P(x < c) = 0.95\]

\[c = \text{invNorm}(0.95, 80, 5) = 88.2243\]

d. The middle 60% of values are between which 2 numbers?

\[P(c_1 < x < c_2) = 0.60\]

We need to break this problem into 2 parts

\[P(x < c_1) = 0.20\]

\[c_1 = \text{invNorm}(0.20, 80, 5) = 75.7919\]

\[P(x < c_2) = 0.80\]

\[c_2 = \text{invNorm}(0.80, 80, 5) = 84.2081\]