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We want time derivatives so taking \( \frac{d}{dt} \) of both sides:  

\[
\frac{dV}{dt}(V) = \frac{d}{dt} \left( \frac{4}{3} \pi r^3 \right)
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The chain rule gives us:

$$\frac{dV}{dt}(V) = \frac{d}{dt}\left(\frac{4}{3}\pi r^3\right)$$

$$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$$
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Solving:

So \( \frac{dr}{dt} = \frac{30}{400 \pi} \approx 0.024 \text{ cm/sec} \)
Related Rates Problems

The draining cone.

Water is draining out of an inverted cone (with radius 10 ft. and height 25 ft) at a rate of 0.5 ft³/min. How quickly is the depth of the water changing when the water is 8 feet deep?
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Then \( V = \frac{\pi}{3} (\frac{2}{5} h)^2 \cdot h \)

\[
= \frac{4\pi}{75} h^3
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Then taking time derivatives implicitly gives us \( \frac{dV}{dt} = \frac{4\pi}{25} h^2 \cdot \frac{dh}{dt} \)
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Then taking time derivatives implicitly gives us \( \frac{dV}{dt} = \frac{4\pi}{25} h^2 \cdot \frac{dh}{dt} \)

So \(-0.5 = \frac{4\pi}{25} (8)^2 \cdot \frac{dh}{dt} \quad \rightarrow \quad \frac{dh}{dt} = -\frac{25}{512\pi} \) ft/min.

Or dropping by a little over \( \frac{3}{16}'' \) per minute.
Related Rates Problems

The speed trap.

There’s this cop with a LIDAR unit . . .
Related Rates Problems

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There’s this cop with a radar unit . . .
And he’s sitting in his car 40 feet off the main road targeting cars moving towards him.
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\[
\frac{dr}{dt} = 100 \text{ ft/sec}
\]

When he pulls you over he claims that you were actually traveling about 70 mph on the road.
Is this possible?!