Linear Functions in $\mathbb{R}^3$

(1.) Sketch the graph (trace) of $6x + 5y - 3z = 30$. 
Linear Functions in $\mathbb{R}^3$

(1.) Sketch the graph (trace) of $6x + 5y - 3z = 30$.

<table>
<thead>
<tr>
<th>$x$</th>
<th>$y$</th>
<th>$z$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>___</td>
</tr>
<tr>
<td>0</td>
<td>___</td>
<td>0</td>
</tr>
<tr>
<td>___</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

(You can fill in the missing values based on the equation and graph.)
Linear Functions in $\mathbb{R}^3$

(1.) Sketch the graph (trace) of $6x + 5y - 3z = 30$.

<table>
<thead>
<tr>
<th>$x$</th>
<th>$y$</th>
<th>$z$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>-10</td>
</tr>
<tr>
<td>0</td>
<td>6</td>
<td>0</td>
</tr>
<tr>
<td>5</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>
Note, if we solve $6x + 5y - 3z = 30$ for $z$ we have

$$z = f(x, y) = 2x + \frac{5}{3}y - 10$$
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In the $xz$–plane we have a slope of $\frac{\Delta z}{\Delta x} = \frac{2}{1}$ and in the $yz$–plane we have a slope of $\frac{\Delta z}{\Delta y} = \frac{5}{3}$. 
Example: Find an equation for the linear function with the table below.

<table>
<thead>
<tr>
<th></th>
<th>10</th>
<th>20</th>
<th>30</th>
<th>40</th>
</tr>
</thead>
<tbody>
<tr>
<td>100</td>
<td>3</td>
<td>6</td>
<td>9</td>
<td>12</td>
</tr>
<tr>
<td>200</td>
<td>2</td>
<td>5</td>
<td>8</td>
<td>11</td>
</tr>
<tr>
<td>300</td>
<td>1</td>
<td>4</td>
<td>7</td>
<td>10</td>
</tr>
<tr>
<td>400</td>
<td>0</td>
<td>3</td>
<td>6</td>
<td>9</td>
</tr>
</tbody>
</table>
Example: Find an equation for the plane containing $(1, 2, 3), (4, -1, -2)$, and $(-3, 1, 1)$. 
**Example:** Find an equation for the plane containing \((1, 2, 3), (4, -1, -2), \) and \((-3, 1, 1)\).

**Solution:**
We want a solution of the form \(z = mx + ny + z_0\) so substituting into the equation gives us:

\[
\begin{align*}
3 &= m + 2n + z_0 \quad (1) \\
-2 &= 4m - n + z_0 \quad (2) \\
1 &= -3m + n + z_0 \quad (3)
\end{align*}
\]

The difference of (1) and (2) gives \(5 = -3m + 3n\) and the difference of (2) and (3) gives \(-3 = 7m - 2n\) this produces:

\[
\begin{align*}
5 &= -3m + 3n \quad (4) \\
-3 &= 7m - 2n \quad (5)
\end{align*}
\]

Multiplying (4) by 2 and (5) by 3 gives

\[
\begin{align*}
10 &= -6m + 6n \quad (6) \\
-9 &= 21m - 6n \quad (7)
\end{align*}
\]

Solving gives us \(m = \frac{1}{15}\)
Since $m = \frac{1}{15}$ it follows

$$5 = -3 \left( \frac{1}{15} \right) + 3n \longrightarrow n = \frac{26}{15}$$

And $3 = \frac{1}{15} + 2 \left( \frac{26}{15} \right) + z_0 \longrightarrow z_0 = -\frac{8}{15}$

Then we have $z = \frac{1}{15} x + \frac{26}{15} y - \frac{8}{15}$