In this chapter we have focused on first-order autonomous systems of differential equations, such as the predator-prey systems described in Section 2.1. In particular, we have seen how such systems can be studied using vector fields and phase plane analysis and how solution curves in the phase plane relate to the \( x(t) \)- and \( y(t) \)-graphs of the solutions. In this lab project you will use these concepts and related numerical computations to study the behavior of the solutions to two different systems.

We have discussed predator-prey systems at length. These are systems in which one species benefits while the other species is harmed by the interaction of the two species. In this lab you will study two other types of systems—competitive and cooperative systems. A competitive system is one in which both species are harmed by interaction, for example, cars and pedestrians. A cooperative system is one in which both species benefit from interaction, for example, bees and flowers. Your overall goal is to understand what happens in both systems for all possible nonnegative initial conditions. Several pairs of cooperative and competitive systems are given at the end of this lab. (Your instructor will tell you which pair(s) of systems you should study.) The analytic techniques that are appropriate to analyze these systems have not been discussed so far, so you will employ mostly geometric/qualitative and numeric techniques to establish your conclusions. Since these are population models, you need consider only \( x \) and \( y \) in the first quadrant (\( x \geq 0 \) and \( y \geq 0 \)).
Your report should include:

1. A brief discussion of all terms in each system. For example, what does the coefficient to the $x$ term in equation for $dx/dt$ represent? Which system is cooperative and which is competitive?

2. For each system, determine all relevant equilibrium points and analyze the behavior of solutions whose initial conditions satisfy either $x_0 = 0$ or $y_0 = 0$. Determine the curves in the phase plane along which the vector field is either horizontal or vertical. Which way does the vector field point along these curves?

3. For each system, describe all possible population evolution scenarios using the phase portrait as well as $x(t)$- and $y(t)$-graphs. Give special attention to the interpretation of the computer output in terms of the long-term behavior of the populations.

Your report: The text of your report should address the three items above, one at a time, in the form of a short essay. You should include a description of all “hand” computations that you did. You may include a limited number of pictures and graphs. (You should spend some time organizing the qualitative and numerical information since a few well-organized figures are much more useful than a long catalog.)
For TBA project #2 (Text Lab 2.2), analyze System Pair #4 in the following table. If you work on the wrong system pair you will receive no credit. In addition, the projects are due the evening of the final exam, no late TBA project work will be accepted.

Remember, as always, I am grading these TBA projects based on:
1. Completeness;
2. Correctness;
3. Clarity
Systems:

Pair (1):

A. \( \frac{dx}{dt} = -5x + 2xy \)  
   \( \frac{dy}{dt} = -4y + 3xy \)

B. \( \frac{dx}{dt} = 6x - x^2 - 4xy \)  
   \( \frac{dy}{dt} = 5y - 2xy - 2y^2 \)

Pair (2):

A. \( \frac{dx}{dt} = -3x + 2xy \)  
   \( \frac{dy}{dt} = -5y + 3xy \)

B. \( \frac{dx}{dt} = 5x - x^2 - 3xy \)  
   \( \frac{dy}{dt} = 8y - 3xy - 3y^2 \)

Pair (3):

A. \( \frac{dx}{dt} = -4x + 3xy \)  
   \( \frac{dy}{dt} = -3y + 2xy \)

B. \( \frac{dx}{dt} = 5x - 2x^2 - 4xy \)  
   \( \frac{dy}{dt} = 7y - 4xy - 3y^2 \)

Pair (4):

A. \( \frac{dx}{dt} = -5x + 3xy \)  
   \( \frac{dy}{dt} = -3y + 2xy \)

B. \( \frac{dx}{dt} = 9x - 2x^2 - 4xy \)  
   \( \frac{dy}{dt} = 8y - 5xy - 3y^2 \)